ANTENNAS AND WAVE PROPAGATION
(3-1 ECE, R19, JNTUA)

Prepared By

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UNIT-I(ANTENNA CHARACTERISTICS)

1. DEFINITION OF ANTENNA:

Antenna is defined as follows: Antenna is basically a wire or current carrying conductor.
(1) A device which converts electrical signal into Electromagnetic waves or vice versa.
(2) A transitional device or transducer which converts electrical energy into EM wave energy
(3) A device which converts single dimensional singnal into three dimensional signal
Consider a Communication system shown in the following figure.

2. RADIATION MECHNAISM OF WIRE ANTENNAS AND CURRENT DISTRIBUTION:

2.1 Radiation Mechanism of single-wire antenna:

Conducting wires are material whose prominent characteristic is the motion of electric charges and the creation of current flow. Let us assume that an electric volume charge density, represented by q_v(coloumbs/m^3), is distributed uniformly in a circular wire of cross sectional area 'A' and volume 'V', as shown in figure below.

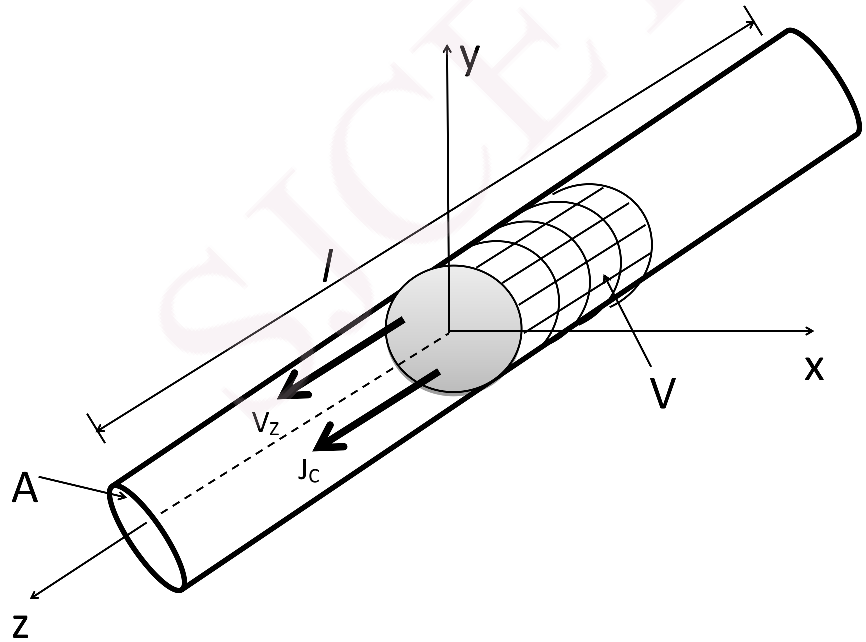


Figure 1: Charge uniformly distributed in a circular cross section cylinder

The total charge Q within volume V is moving in the z direction with a uniform velocity v_z(meters/sec). The current density J_z(ampere/m^2) over the cross section of the wire is given by

$$J_z = q_v v_z \qquad -(1)$$

If the wire is made of an ideal electric conductor, the current density J_s(ampere/m) resides on the surface of the wire and it is given by

$$J_s = q_s v_z \qquad -(2)$$

Where q_s (coulombs/m^2) is the surface charge density. If the wire is very thin(ideally zero radius), Then the current in the wire can be represented by

$$I_z = q_l v_z \qquad -(3)$$

Where q_l (coulombs/m) is the line charge density.
Instead of examining all three current densities, we will primarily concentrate on the very thin wire. The conclusions apply to all three. If the current is time-varying then the current of equation 3 can be written as

$$\frac{dI_z}{dt} = q_l \frac{dv_z}{dt} = q_l a_z \qquad -(4)$$

Where $dv_z/dt = a_z$ (meters/sec^2) is the acceleration. If the wire is of length 'l', then equation 4 can be written as

$$l\frac{dI_z}{dt} = lq_l \frac{dv_z}{dt} = lq_l a_z \qquad -(5)$$

From equation 5, the following points can be observed:
(i) If a charge is not moving, current is not created and there is no radiation.
(ii) If charge is moving with a uniform velocity:
(a) There is no radiation if the wire is straight, and infinite in extent
(b) There is radiation if the wire is curved, bent, discontinuous, terminated, or truncated as shown in figure 2.
(iii) If charge is oscillating in a time-motion, it radiates even if the wire is straight.

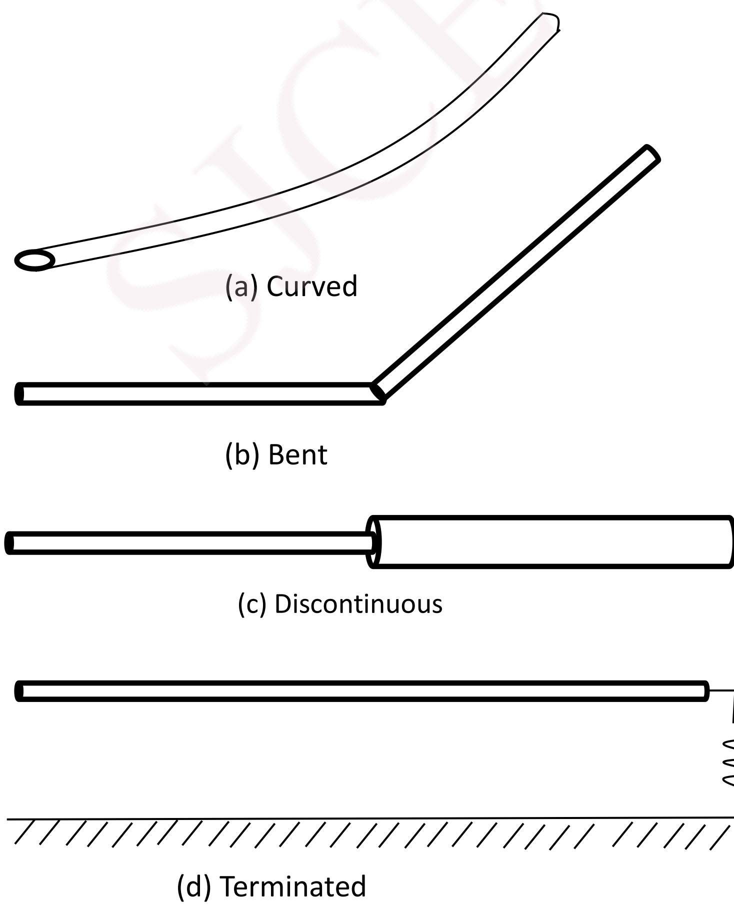


Figure 2: Wire configuration for radiation

2.2 Radiation Mechanism of two-wire antenna:

The two wire transmission line can also act as an antenna. To understand this let us consider the two-wire transmission line which is shown in the figure below.

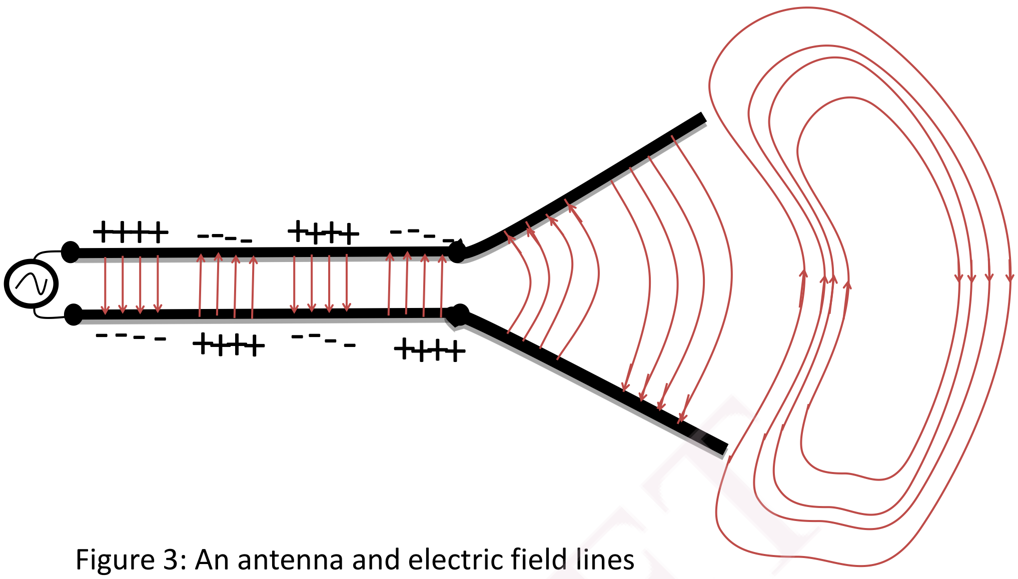


Figure 3: An antenna and electric field lines

The transmission line act as an antenna when its second port is open circuited. In case of transmission line the EM waves will present in between the two wires as shown on the figure. These EM waves will enter in to the free space through the open circuit in the form of radiation. To have the impedance matching between the transmission line and the free space, and to avoid the diffraction, the transmission line is tapered at the second port. Therefore the antenna is a transition device or transducer between a guided wave and the free space wave or vice versa.

2.3 Current Distribution:

In earlier sections we discussed the movement of the free electrons on the conductors representing the transmission line and the antenna. In order to illustrate the creation of the current distribution on a linear dipole, and its subsequent radiation, let us first begin with geometry of a lossless two-wire transmission line, as shown in figure 4(a). The movement of the charges creates a traveling wave current, of magnitude $I_0/2$, along each of the wires. When the current arrives at the end of each of the wires, it undergoes a complete reflection(equal magnitude and 180^0 phase reversal). The reflected traveling wave, when combined with the incident traveling wave, forms in each wire a pure standing wave pattern of sinusoidal form as shown in figure 4(a). The current in each wire undergoes a 180^0 phase reversal between adjoining half cycles. This is indicated in figure 4(a) by the reversal of the arrow direction. Radiation from each wire individually occurs because of the time-varying nature of the current and the termination of the wire.

For the two-wire balanced(symmetrical) transmission line, the current in a half cycle of one wire is of the sae magnitude but 180^0 out-of-phase from that in the corresponding half-cycle of the other wire. If in addition the spacing between the two wires is very small($s \ll \lambda$), the fields radiated by the current of each wire are essentially cancelled by those of the other. The net result is an almost ideal nonradiating transmission line.

As the section of the transmission line between $0 \leq z \leq l/2$ begins to flare, as shown in figure 4(b), it can be assumed that the current distribution is essentially unaltered in form in each of the wires. However, because the two wires of the flared section are not necessarily close to each other, the fields radiated by one do not necessarily cancel those of the other. Therefore ideally there is a net radiation by the transmission line system.

Ultimately the flared section of the transmission line can take the form shown in figure 4(C). This is the geometry of the widely used dipole antenna. Because of the standing wave current pattern, it is also classified as a standing wave antenna.

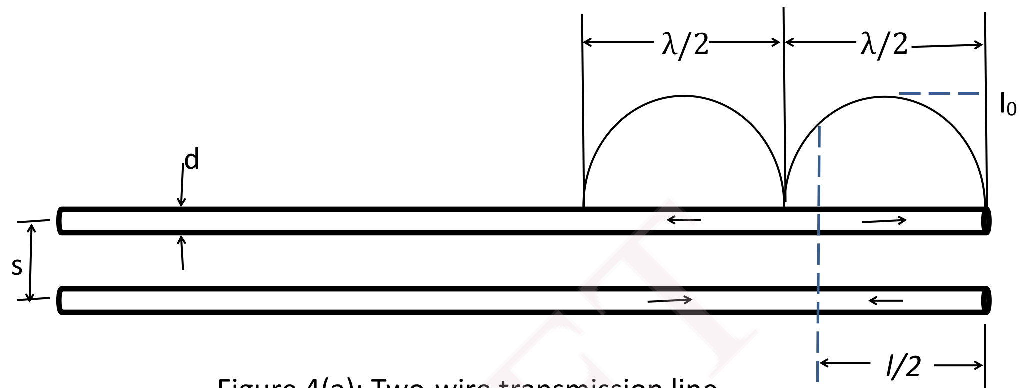


Figure 4(a): Two-wire transmission line

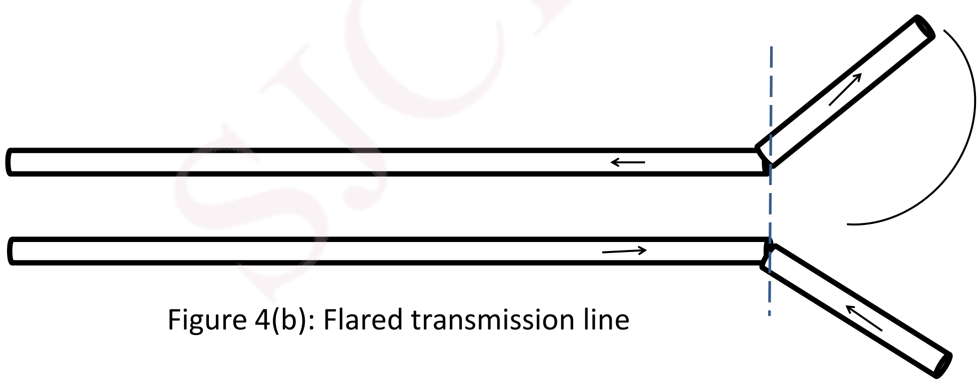


Figure 4(b): Flared transmission line

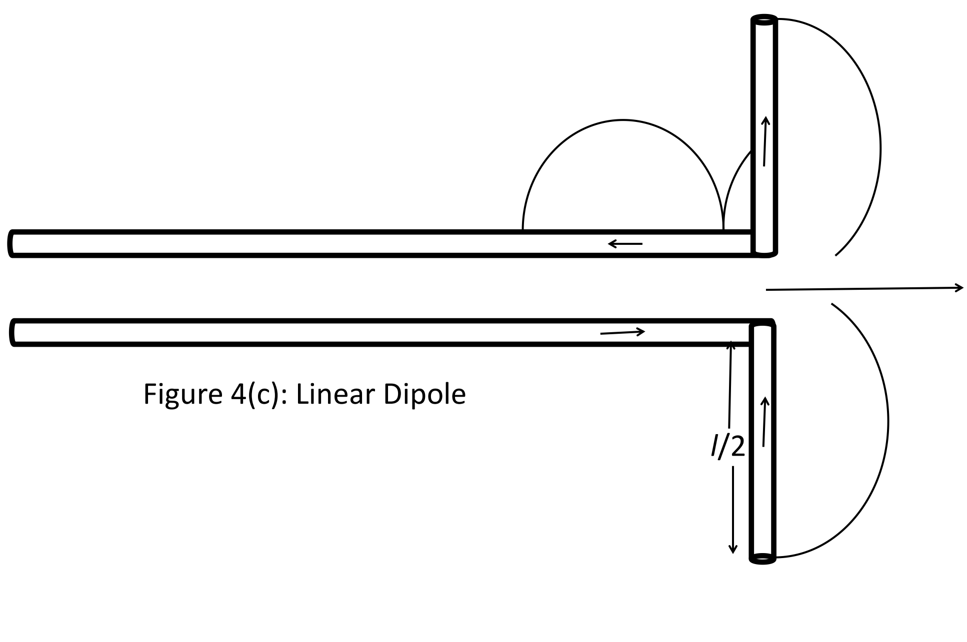


Figure 4(c): Linear Dipole

3. ISOTROPIC RADIATOR:

An isotropic radiator is defined as an antenna which radiates equally in all directions. It is also called as the Isotropic source or omnidirectional radiator or simply unipole. The following are the important characteristics of isotropic radiator.

i. It is the lossless antenna
ii. The gain or directivity of isotropic radiator is unity
iii. It is used as the standard or reference antenna to compare all the practical antennas.
iv The radiation pattern of the isotropic radiator is spherical shape or omnidirectional Pattern or broadcasting pattern.

Now let us derive the small equation for the power density radiated by the isotropic radiator. Imagine the isotropic radiator is located at the centre of the sphere of radius 'r'. Then the energy radiated by the radiator must pass over the surface area of the sphere.

The total power radiated by the isotropic radiator is given by

$$W = \iint P \cdot ds \qquad - (1)$$

Where 'P' represents the poynting vector in watts/m^2

But the waves generated by any radiator will travel in the radius direction. i.e.r-direction. Then the poynting vector can be represented with P_r.

Then

$$W = \iint P_r \cdot ds \qquad - (2)$$

But $ds = r^2 \sin\theta \, d\theta d\emptyset$ in spherical coordinate system in the r-direction.

$$W = \iint P_r \cdot r^2 \sin\theta \, d\theta d\emptyset$$

$$W = P_r \iint r^2 \sin\theta \, d\theta d\emptyset$$

$$W = P_r r^2 \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\emptyset$$

$$W = P_r r^2 [-\cos\theta]_0^{\pi} [\emptyset]_0^{2\pi}$$

$$W = P_r r^2 [2][2\pi]$$

$$W = 4\pi r^2 P_r$$

$$P_r = \frac{W}{4\pi r^2} \quad \text{watts/m}^2 \qquad - (3)$$

The above equation represents the power density radiated by the isotropic radiator. The letter P_r is called the radial component of the poynting vector.

4. RADIATION PATTERN:

The radiation pattern is defined as the graphical representation of radiation properties with respect to the space coordinates. The radiation properties are radiation intensity, field strength and phase or polarization. When the radiation pattern is expressed in terms of the field strength then it is called field pattern or when it is expressed in terms of power then it is called the power pattern. The radiation pattern place very important role in analyzing the performance of any practical antenna.

The examples of radiation patterns are

(i) Omnidirectional or broadcasting pattern

(ii) Pencil beam pattern

(iii) Fan beam pattern

(iv) Shaped beam pattern

In addition to these, there are other pattern shapes like Limacon, Cardioid, figure of eight or doughnut shape. The radiation pattern produced by the dipole is shown in the following figure1.1.

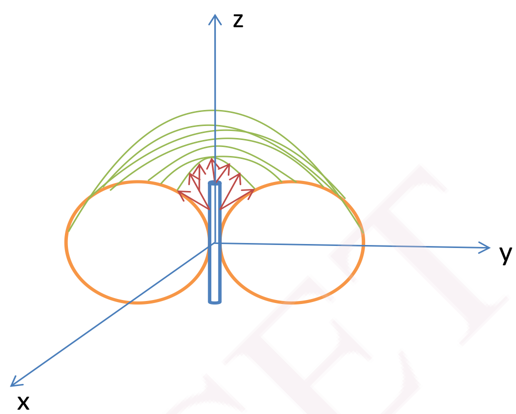


Fig1.1a: Half of the three dimensional pattern (Doughnut shape)

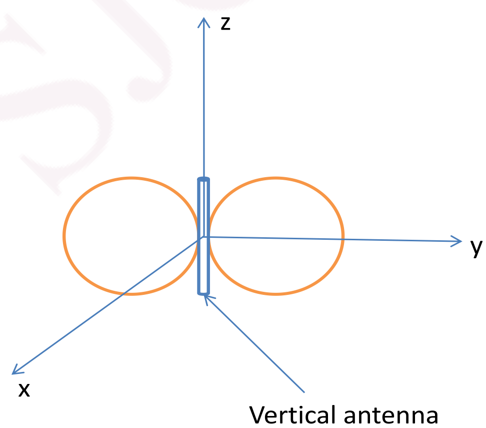


Fig1.1b: Two dimensional pattern obtained by cutting the three dimensional pattern with vertical plane along the axis of the dipole.

5. PATTERNS IN THE PRINCIPAL PLANES:

The performance of any antenna can be described in terms of its patterns in the principal planes. Basically there are two types of patterns in the principal plane such as E-plane principal pattern and H-plane principal pattern. These two patterns are shown in the figure1.3 below.

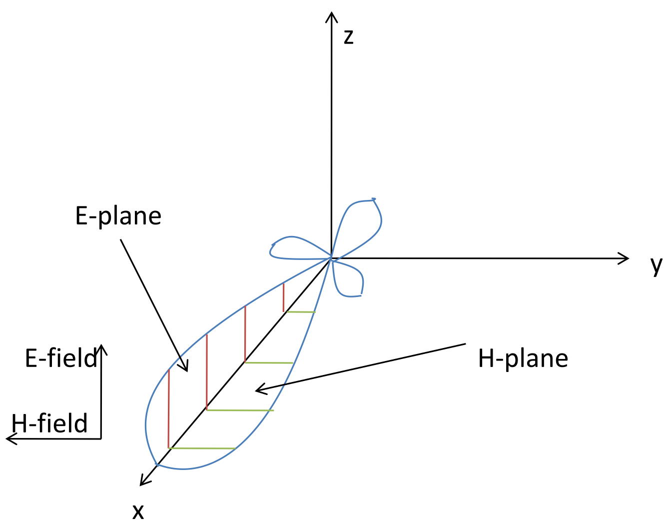


Fig1.3: E-plane and H-plane principal patterns

The radiation pattern of any antenna contains two planes such as E-plane and H-plane. The E-plane principal pattern is defined as the plane containing the electric field vector and direction of maximum radiation. Similarly the H-plane pattern is defined as the plane containing magnetic field vector and the direction of maximum radiation.

6. <u>RADIATION PATTERN LOBES:</u>

Different parts of radiation pattern are referred as the lobes. Various lobes are shown in the figure 1.4 and figure1.5 below.

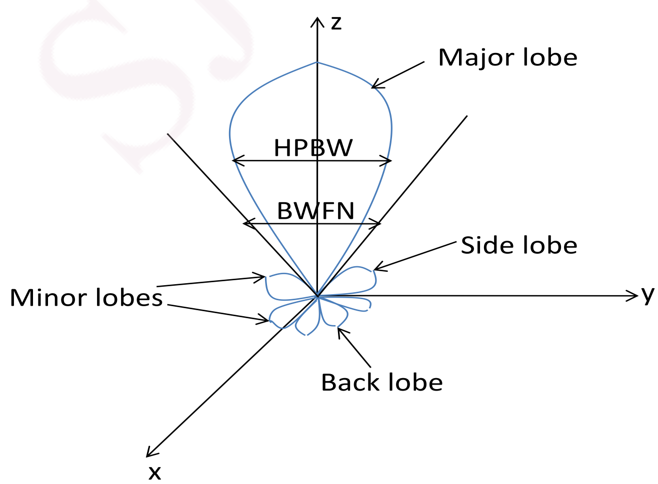


Fig1.4: Radiation pattern lobes (three dimensional)

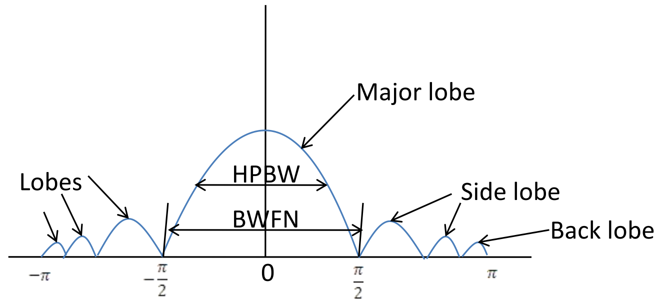


Fig1.5: Linear plot of radiation pattern (two dimensional)

Major lobe: A lobe which contains the direction of maximum radiation is called the main lobe or major lobe. The radiation pattern which contains a single major lobe is called the unidirectional radiation pattern. Whereas the radiation pattern which contains two major lobes is called the bidirectional radiation pattern.

Minor lobe: Any lobe except the main lobe is called the minor lobe. That means other than the major lobes the remaining lobes are called the minor lobes. Practically the minor lobes should be eliminated to improve the efficiency of any antenna.

Side lobe: It is a minor lobe which is existing in any direction other than the intended direction. Normally the side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main lobe.

Back lobe: It is a minor lobe which occupies the hemisphere in a direction exactly opposite to the main lobe.

Minor lobes are the small radiation lobes which represents the radiation in the undesired direction. In the figures 1.4 and 1.5, the terms HPBW represents the Half Power Beam Width whereas BWFN represents the Beam Width between the First Nulls. These two terms will be defined in the later sections.

7. RADIAN AND STERADIAN:

The unit of plane angle is radian. The radian is defined as the plane angle with its vertex at the centre of a circle of radius ‘r’ that is subtended by an arc whose length is ‘r’ as shown in the figure1.6 below.

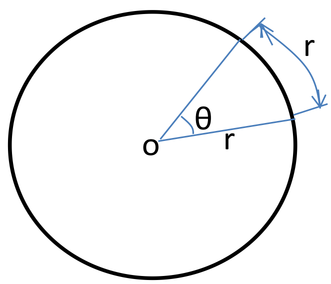


Fig1.6: Radian

The number of radians in a complete circle is given by

Total no.of radians $= \frac{Circumference\ of\ circle}{arc\ length} = \frac{2\pi r}{r} = 2\pi$

Therefore in the total number of radians in a complete circle is 2π.

Similarly the unit of solid angle is steradian. The steradian is defined as the solid angle with its vertex at the centre of sphere of radius ‘r’ that is subtended by the spherical surface of area which is equal to the area of square with each side of lenght ‘r’ as shown in the figure1.7 below.

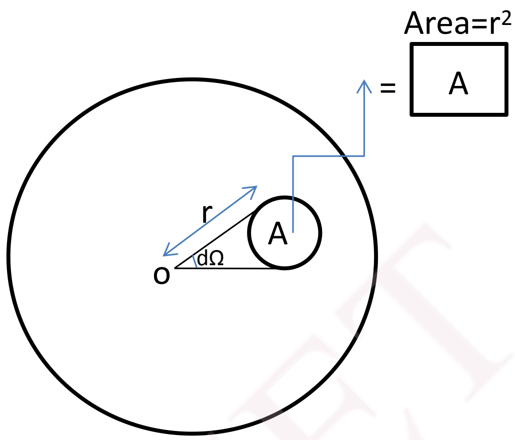


Fig1.7: Steradian

The total number of steradians in a complete sphere is given by

Total no.of steradians $= \frac{\text{Area of the sphere}}{\text{differential surface area}} = \frac{4\pi r^2}{r^2} = 4\pi$

$$d\Omega = 4\pi$$

Therefore in a complete sphere there are 4π steradians. The number of steradians in a sphere along the r-direction is given by

$$d\Omega = \frac{ds}{r^2} \qquad - (1)$$

But $ds = r^2 \sin\theta d\theta d\emptyset$

Therefore $$d\Omega = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset \qquad - (2)$$

8. <u>RADIATION INTENSITY:</u>

The radiation intensity is defined as the power per unit solid angle. The radiation intensity is represented with ϕ. Therefore

$$\Phi = \frac{\text{power}}{\text{unit solid angle}} \qquad - (1)$$

The unit of radiation intensity is watts/steradian.

We know that the poynting vector is

$$P = E \times H \text{ watts/m}^2 \qquad - (2)$$

The average poynting vector is given by

$$P_{av} = \frac{1}{2}(E \times H^*) \qquad - (3)$$

But $\frac{E}{H} = \boldsymbol{\eta}$

The pointing vector becomes

$$P = \frac{E^2}{\eta} \qquad - (4)$$

And the average pointing vector or radial component of pointing vector becomes

$$P_{av} = P_r = \frac{1}{2}\frac{E^2}{\eta} \qquad - (5)$$

The radiation intensity in terms of P_r is given by

$$\Phi = P_r \cdot r^2 \qquad - (6)$$

Since $r^2 = \frac{ds}{d\Omega}$

Substitute equation 5 in equation 6

$$\Phi = \frac{1}{2}\frac{E^2(\theta,\emptyset)}{\eta} \cdot r^2 \text{ watts/steradian}$$

The radiation intensity can also defined as

$$\Phi = \frac{\text{differential power}}{\text{differential element of solid angle}} = \frac{dW_r}{d\Omega}$$

$$\Phi = \frac{dW_r}{d\Omega} \qquad - (7)$$

$$dW_r = \phi d\Omega$$

Take integration on both sides

$$\int dW_r = \int \phi d\Omega$$

$$W_r = \int \phi d\Omega \qquad - (8)$$

The above equation represents the power radiated by the antenna. In case of isotropic radiator the solid angle is 4π (because the shape of the radiation pattern due to the isotropic radiator is sphere and hence the 4π is total steradians present in the sphere) and the radiated power becomes

$$W_r = \int 4\pi\phi$$

$$W_r = 4\pi \int \phi$$

$$W_r = 4\pi\phi_{av}$$

Where $\phi_{av} = \int \phi$

Therefore $\phi_{av} = \frac{W_r}{4\pi}$ - (11)

The above equation is called the average radiation intensity.

9. <u>GAIN(G):</u>

In general the terms like Gain, Directive gain, Power gain, Directivity having the similar meaning. Theoretically these terms having slightly different definitions but practically all the terms are same. The gain or directivity Gain or directivity or power gain is represents the ability of transmitting antenna to radiate in to a certain direction or the ability of the receiving antenna to receive the signal from the certain direction.

Specifically the gain will be defined as follows:

$$\text{Gain(G)} = \frac{\text{Maximum radiation intensity from the test antenna}}{\text{Maximum radiation intensity from the reference antenna}}$$

And

$$\text{Gain(G)} = \frac{\text{Maximum power received by the test antenna}}{\text{Maximum power received by the reference antenna}}$$

10. DIRECTIVE GAIN(G_d):

It is the ability of an antenna to concentrate more power in a preferred direction. The directive gain is denoted with G_d. The directive gain will be defined in the following ways;

$$G_d = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}}$$

$$G_d = \frac{\phi(\theta, \emptyset)}{\phi_{av}}$$

But

$$\phi_{av} = \frac{W_r}{4\pi}$$

Therefore

$$G_d = \frac{\phi(\theta, \emptyset)}{\frac{W_r}{4\pi}} = \frac{4\pi\phi(\theta, \emptyset)}{W_r}$$

$$G_d = \frac{4\pi\phi(\theta, \emptyset)}{W_r} \qquad \text{- (1)}$$

Also the directive gain will be defined as

$$G_d = \frac{\text{Power density from the test antenna}}{\text{power density from the reference antenna}}$$

Generally the isotropic radiator will be used as the reference antenna. The directive gain does not include the losses of an antenna.

11. POWER GAIN(G_p):

The power gain is similar to the directive gain except the power gain includes the losses of an antenna where as the directive gain do not includes the losses of an antenna. The power gain is denoted with G_p. The power gain will be defined as follows:

$$G_P = \frac{\text{Power density from the test antenna}}{\text{Power density from thr reference antenna}}$$

The power gain is also defined as

$$G_P = \frac{\text{Radiation intensity in a given direction}}{\text{Average radiation intensity}}$$

$$G_P = \frac{\phi(\theta, \emptyset)}{\frac{W_T}{4\pi}} = \frac{4\pi\phi(\theta, \emptyset)}{W_T} \qquad \text{- (1)}$$

Where W_T is the total power applied to the antenna

$$W_T = W_r + W_l$$

Where W_l is the loss power.

The power gain also defined as

$$G_P = \frac{\text{Power input supplied to the test antenna}}{\text{Power input supplied to the reference antenna}}$$

The relation in between the directive gain and power gain is given by

$$G_P = \eta G_d \qquad - (2)$$

12. DIRECTIVITY (D):

The maximum directive gain is nothing but directivity. It is denoted with D. The directivity is defined in the following ways:

$$D = \frac{\text{Maximum radiation intensity from the test antenna}}{\text{Average radiation intensuty from the test antenna}}$$

$$D = \frac{\text{Maximum radiation from the test antenna}}{\text{Radiation intensity from the reference antenna}}$$

$$D = \frac{\text{Power radiated from the test antenna}}{\text{Power radiated from the reference antenna}}$$

13. ANTENNA EFFICIENCY:

The antenna efficiency is defined as the ratio of radiated power to the power input supplied to the antenna. That is

$$\eta = \frac{\text{Power radiated}}{\text{Power input to the antenna}}$$

$$\eta = \frac{W_r}{W_T} = \frac{W_r}{W_r + W_l} \qquad - (1)$$

When the loss power(W_l) is neglected , then the antenna efficiency will be 100%.

Multiply numerator and denominator of equation 1 with $\phi(\theta,\varphi)$

$$\eta = \frac{W_r}{W_T} \times \frac{4\pi\phi(\theta,\emptyset)}{4\pi\phi(\theta,\emptyset)} = \frac{4\pi\phi(\theta,\emptyset)}{W_T} \frac{W_r}{4\pi\phi(\theta,\emptyset)}$$

$$\eta = G_P \cdot \frac{1}{G_d}$$

$$\eta = \frac{G_P}{G_d} \qquad - (2)$$

The antenna efficiency can also expressed in terms of radiation resistance(R_r) and loss resistance(R_l) as follows:

We know that

$$W_r = I^2 R_r \text{ and } W_l = I^2 R_l \qquad - (3)$$

Substitute above equation in equation 1

Then

$$\eta = \frac{W_r}{W_r + W_l} = \frac{I^2 R_r}{I^2 R_r + I^2 R_l} = \frac{I^2 R_r}{I^2 (R_r + R_l)}$$

$$\eta = \frac{R_r}{R_r + R_l} \qquad - (4)$$

14. EFFECTIVE AREA OR EFFECTIVE APERTURE OR CAPTURE AREA:

The effective area or effective aperture or capture area is defined as the ratio of the power received to the poynting vector of the incident field. That is

$$A_e = \frac{\text{Power received}}{\text{Poynting vector of the incident wave}}$$

$$A_e = \frac{W}{P} \ m^2 \qquad - (1)$$

To derive the equation for the effective area let us consider the following figure 1.8

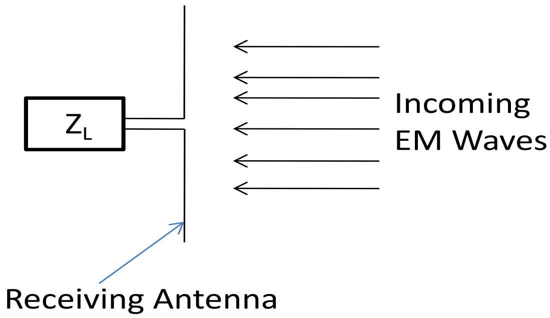


Fig1.8a: Receiving antenna

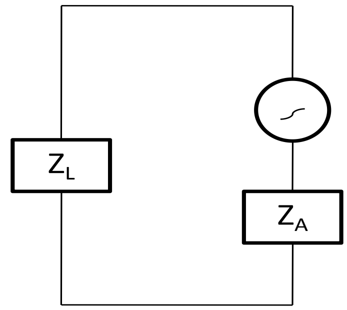


Fig1.8b: Equivalent circuit

Let I be the current flowing in the receiving antenna due to the incident EM waves, then the power received is given by

$$W = I_{rms}^2 R_L \qquad - (2)$$

To satisfy the maximum power transfer theorem only load resistance (instead of load impedance) is considered in the above equation.

Substitute equation 2 in equation 1

$$A_e = \frac{I_{rms}^2 R_L}{P} \qquad - (3)$$

From the equivalent circuit we have

$$I_{rms} = \frac{V}{Z_L + Z_A} \qquad - (4)$$

Where $Z_L = R_L + jX_L$ called the load impedance

And $Z_A = R_A + jX_A$ called the antenna impedance

$$\therefore \quad I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)}$$

$$I_{rms} = \frac{V}{R_L + jX_L + R_A + jX_A}$$

$$I_{rms} = \frac{V}{(R_L + R_A) + j(X_L + X_A)}$$

$$|I_{rms}| = \frac{V}{\sqrt{(R_L+R_A)^2 + (X_L+X_A)^2}} \qquad - (5)$$

Substitute equation 5 in the equation 3

$$A_e = \left[\frac{V}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}}\right]^2 \frac{R_L}{P}$$

$$A_e = \frac{V^2 R_L}{[(R_L+R_A)^2 + (X_L+X_A)^2]P} \qquad - (6)$$

To deliver the maximum power to the load we need to satisfy the maximum power transfer theorem. According to the maximum power transfer theorem the source impedance and load impedance must be complex conjugates to each other. i.e

$$R_L = R_A \qquad - (7)$$

And $$X_L = - X_A \qquad - (8)$$

But $R_A = R_r + R_l = R_r$ when R_l is neglected.

Therefore $$R_L = R_r \qquad - (9)$$

Substitute equations 7 and 8 in equation 6

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2 + (X_L - X_L)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2 + (0)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(R_L + R_L)^2]P}$$

$$A_{em} = \frac{V^2 R_L}{[(2R_L)^2]P} = \frac{V^2 R_L}{4PR_L^2}$$

$$A_{em} = \frac{V^2}{4PR_L} \qquad \textit{- (10)}$$

In addition to the above effective area we need to define some more like Scattering aperture, loss aperture, collecting aperture and physical aperture.

When the effective area is defined in terms of radiation resistance(R_r) then it is called as the Scattering aperture(A_s) and is given by

$$\mathrm{A_s} = \frac{\mathrm{V}^2}{4\mathrm{PR_r}} \qquad \text{- (11)}$$

When the effective area is defined in terms of loss resistance(R_{rl}) then it is called as the Loss aperture(A_l) and is given by

$$\mathrm{A_s} = \frac{V^2}{4PR_l} \qquad \text{- (12)}$$

The ration of loss aperture and effective aperture is called the effectiveness ratio and is given by

$$\alpha = \frac{A_l}{\mathrm{A_{em}}} \qquad \text{- (13)}$$

The collecting aperture(A_c) is nothing but a collection of all the three apertures such as effective aperture, scattering aperture and loss aperture. i.e.

$$A_c = A_e + A_s + A_l \qquad \text{- (14)}$$

The ration of scattering aperture to the effective aperture is nothing but scattering ratio and is given by

$$\beta = \frac{\mathrm{A_s}}{\mathrm{A_e}} \qquad \text{- (15)}$$

The Physical aperture is nothing but physical area of the antenna which depends upon the physical dimensions of the antenna. The ratio in between the maximum effective area and the physical area is nothing but a absorption ratio or aperture efficiency and is given by

$$\gamma = \varepsilon_{ap} = \frac{A_{em}}{A_p} \qquad \text{- (16)}$$

15. <u>Relation in between the effective area and gain or directivity:</u>

Let us consider two antennas A and B and then

D_a be the directivity of antenna A

D_b be the directivity of antenna B

A_{ema} be the maximumveffective area of the antenna A

A_{emb} be the maximum effective area of the antenna B

Practically it is found that the directivity is directly proportional to the effective area. i.e

$$D_a \, \alpha \, A_{ema}$$

and $D_b \alpha A_{emb}$

Take the ratio of the above two equations then

$$\frac{D_a}{D_b} = \frac{A_{ema}}{A_{emb}} \qquad - (1)$$

Let the antenna A be the isotropic radiator, then the gain or directivity of isotropic radiator is unity. i.e. $D_a=1$.

Then above equation becomes

$$\frac{1}{D_b} = \frac{A_{ema}}{A_{emb}} \qquad - (2)$$

$$A_{ema} = \frac{A_{emb}}{D_b} \qquad - (3)$$

$$D_b = \frac{A_{emb}}{A_{ema}} \qquad - (4)$$

Let the antenna B be the short dipole, then the gain or directivity of short dipole is 3/2 and maximum effective area is $\frac{3}{8\pi}\lambda^2$

i.e. $D_b = 3/2$ and $A_{emb} = \frac{3}{8\pi}\lambda^2$

Substitute above two values in the equation 3 then

$$A_{ema} = \frac{\frac{3}{8\pi}\lambda^2}{3/2}$$

$$A_{ema} = \frac{\lambda^2}{4\pi} \qquad - (5)$$

Substitute equation 5 in the equation 4

$$D_b = \frac{A_{emb}}{\frac{\lambda^2}{4\pi}}$$

$$D_b = \frac{4\pi A_{emb}}{\lambda^2}$$

In general, $$D = G = \frac{4\pi A_e}{\lambda^2} \qquad - (6)$$

The above equation gives the relation in between the effective area and the gain or directivity

16. <u>VECTOR EFFECTIVE LENGTH OR EFFECTIVE HEIGHT:</u>

Generally the effective height or effective length represents how far the antenna is involved either in receiving or transmitting the signal. The effective height is defined separately for receiving antenna and transmitting antenna.

For receiving antenna: In case of receiving antenna the effective height or effective length (l_e) is defined as the ratio of the induced voltage and incident electric field strength. i.e.

$$l_e = \frac{\text{Voltage induced}}{\text{Incident electric field strength}} \quad \text{m or } \lambda$$

$$l_e = \frac{V}{E} \qquad - (1)$$

We know that $$A_{em} = \frac{V^2}{4PR_L} \qquad - (2)$$

Also we know that the Poynting vector is $P = E \times H$

Or $$P = \frac{E^2}{\eta} \qquad - (3)$$

Substitute equation 3 in equation 2

$$A_{em} = \frac{V^2}{4\frac{E^2}{\eta}R_L}$$

$$\frac{V^2}{E^2} = \frac{4A_{em}R_L}{\eta}$$

$$l_e = \frac{V}{E} = \sqrt{\frac{4A_{em}R_L}{\eta}}$$

$$l_e = 2\sqrt{\frac{A_{em}R_L}{\eta}} \qquad - (4)$$

$$l_e^2 = \frac{4A_{em}R_L}{\eta}$$

Or

$$l_e^2 = \frac{4A_eR_L}{\eta}$$

$$A_e = \frac{l_e^2\eta}{4R_L} \qquad - (5)$$

For transmitting antenna: In case of transmitting antenna the effective height or effective length is simply the physical length of the antenna where the current is uniform. Theeffective length and physical length of transmitting antenna is represented in the figure 1.9 below.

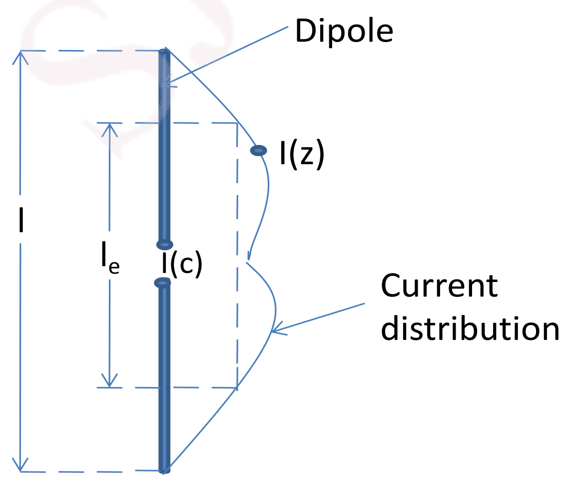


Fig 1.9: Illustration of effective length for transmitting antenna

The meaning of different letters indicated in the above figure are given by
I(c) = Current at the terminals of the actual antenna, I(z) = Current at any point 'z' of the antenna, l_e = Effective length, l = Physical length.

We know that the current element is Idl, with I is current and dl is the differential length.

Therefore $$I(c)l_{et} = \int_{-l/2}^{l/2} I(z)dz$$

$$l_{et} = \frac{1}{I(c)} \int_{-l/2}^{l/2} I(z)dz$$

$$l_{et} = \frac{2}{I(c)} \int_{0}^{l/2} I(z)dz$$

The above equation represents the effective height of the transmitting antenna.

17. <u>RECIPROCITY(ANTENNA THEOREMS):</u>

All the network theorems can be applied to the field theory or antenna theory. Especially the reciprocity theorem is very useful in antenna theory. The reciprocity theorem has got many applications in the antenna theory. Therefore let us discuss about the reciprocity theorem.

Reciprocity Theorem: The reciprocity theorem can stated as follows: **"**If current 'I_1' applied at antenna no.1 induces a voltage or e.m.f 'E_{21}' in antenna no.2 and current 'I_2' at antenna no.2 induces a voltage or e.m.f 'E_{12}' in antenna no.1 then $E_{12} = E_{21}$ if $I_1 = I_2$".

The above statement can be proved by considering the following figure1.10.

Case (i):

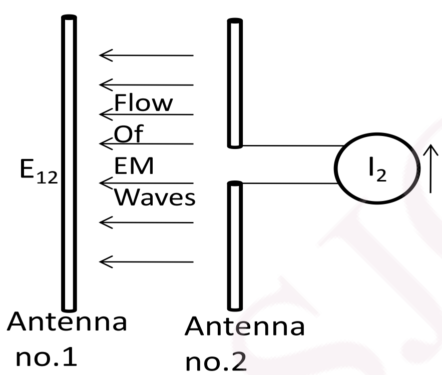


Fig1.10a:Current induces voltage E_{12}

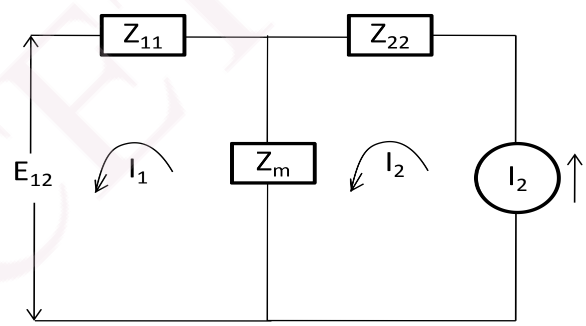


Fig1.10b:Equivalent circuit

In the first case the current I_2 is applied at antenna no.2 and in second case the current I_1 is applied at antenna no.1.

Let I_1= current applied at antenna no.1,

I_2= current applied at antenna no.2,

E_{12} = e.m.f or voltage induced in antenna no.1 due to I_2

E_{21} = e.m.f or voltage induced in antenna no.2 due to I_1

Z_{11} = Self impedance of antenna no.1

Z_{22} = Self impedance of antenna no.2

Z_m = Mutual impedance between the two antennas

Apply KVL to the second loop of figure 1.10b, then

$$I_2(Z_{22} + Z_m) - I_1 Z_m = 0$$

$$I_2(Z_{22} + Z_m) = I_1 Z_m$$

$$I_2 = I_1 \frac{Z_m}{Z_{22}+Z_m} \qquad - (1)$$

Apply KVL to the First loop of figure 1.10b, then

$$I_1(Z_{11} + Z_m) - I_2 Z_m = E_{12} \qquad - (2)$$

Substitute equation 1 in equation 2

$$I_1(Z_{11}+Z_m) - I_1\frac{Z_m}{Z_{22}+Z_m}Z_m = E_{12}$$

$$I_1(Z_{11}+Z_m) - I_1\frac{Z_m^2}{Z_{22}+Z_m} = E_{12}$$

$$I_1\left((Z_{11}+Z_m) - \frac{Z_m^2}{Z_{22}+Z_m}\right) = E_{12}$$

$$I_1\left[\frac{(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2}{Z_{22}+Z_m}\right] = E_{12}$$

$$I_1 = \frac{E_{12}(Z_{22}+Z_m)}{(Z_{11}+Z_m)(Z_{22}+Z_m) - Z_m^2}$$

$$I_1 = \frac{E_{12}(Z_{22}+Z_m)}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m+Z_m^2-Z_m^2}$$

$$I_1 = \frac{E_{12}(Z_{22}+Z_m)}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m} \qquad \text{- (3)}$$

Substitute equation 3 in the equation 1

$$I_2 = \frac{E_{12}(Z_{22}+Z_m)}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m}\left(\frac{Z_m}{Z_{22}+Z_m}\right)$$

$$I_2 = \frac{E_{12}Z_m}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m} \qquad \text{- (4)}$$

Case (ii):

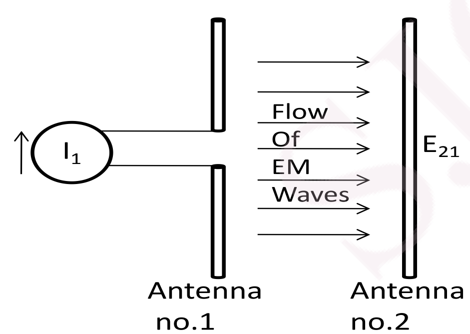


Fig1.11a:Current induces voltage E_{21}

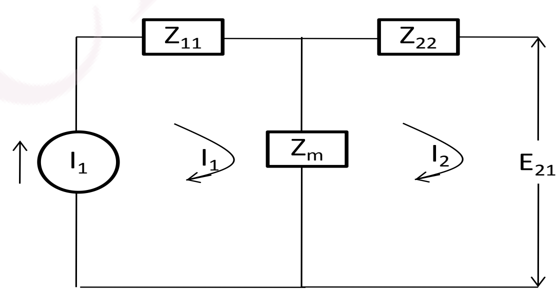


Fig1.11b:Equivalent circuit

Apply KVL to the first loop of figure 1.11b, then

$$I_1(Z_{11}+Z_m) - I_2Z_m = 0$$

$$I_1(Z_{11}+Z_m) = I_2Z_m$$

$$I_1 = I_2\frac{Z_m}{Z_{11}+Z_m} \qquad \text{- (5)}$$

Apply KVL to the second loop of figure 1.11b, then

$$I_2(Z_{22}+Z_m) - I_1Z_m = E_{21} \qquad \text{- (6)}$$

Substitute equation 5 in equation 6

$$I_2(Z_{22}+Z_m) - I_2\frac{Z_m}{Z_{11}+Z_m}Z_m = E_{21}$$

$$I_2(Z_{22}+Z_m) - I_2\frac{Z_m^2}{Z_{11}+Z_m} = E_{21}$$

$$I_2\left((Z_{22}+Z_m)-\frac{Z_m^2}{Z_{11}+Z_m}\right)=E_{21}$$

$$I_2\left[\frac{(Z_{22}+Z_m)(Z_{11}+Z_m)-Z_m^2}{Z_{11}+Z_m}\right]=E_{21}$$

$$I_2=\frac{E_{21}(Z_{11}+Z_m)}{(Z_{22}+Z_m)(Z_{11}+Z_m)-Z_m^2}$$

$$I_2=\frac{E_{21}(Z_{11}+Z_m)}{Z_{22}Z_{11}+Z_{22}Z_m+Z_{11}Z_m+Z_m^2-Z_m^2}$$

$$I_2=\frac{E_{21}(Z_{11}+Z_m)}{Z_{22}Z_{11}+Z_{22}Z_m+Z_{11}Z_m} \qquad \text{- (7)}$$

Substitute equation 7 in the equation 5

$$I_1=\frac{E_{21}(Z_{11}+Z_m)}{Z_{22}Z_{11}+Z_{22}Z_m+Z_{11}Z_m}\left(\frac{Z_m}{Z_{11}+Z_m}\right)$$

$$I_1=\frac{E_{21}Z_m}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m} \qquad \text{- (8)}$$

But $I_1=I_2$

That is equate equations 4 and 8

$$\frac{E_{12}Z_m}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m}=\frac{E_{21}Z_m}{Z_{11}Z_{22}+Z_{11}Z_m+Z_{22}Z_m}$$

$$E_{12}=E_{21}$$

That is the induced voltages are equal when the source (Current) is interchanged.

Applications of Reciprocity theorem:

The main applications of reciprocity theorem are

(i) Equivalence of radiation and receive patterns
(ii) Equivalence of impedances
(iii) Equalities of effective areas
(iv) Equality of antenna directivity
(v) Equality of effective length

Let us discuss the first application. The meaning of the statement "Equality of radiation pattern" is the radiation pattern of any test antenna under the transmitting mode and receiving mode is same. That is the directional pattern of transmitting and receiving antennas are identical if the medium is linear, passive and isotropic then reciprocity theorem holds good. This statement can be proved as follows: As mentioned in the above it is proved that the transmitting and receiving patterns are identical. For this consider the figure 1.12 in which two antennas A1(test antenna) and A2 exploring antenna are shown.

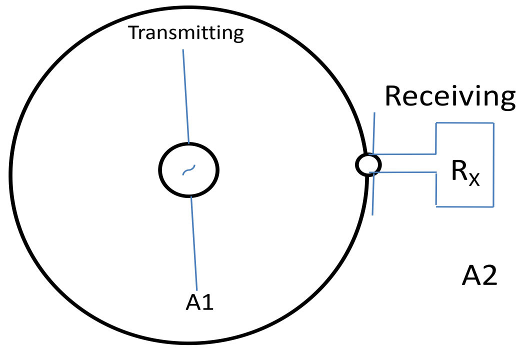


Fig1.12:Measurement of pattern on observation circle

Let A1 be the transmitting antenna and A2 be the receiving antenna. To find out either the field pattern or power pattern the procedure is same. Considering the field pattern, keeping the transmitting antenna no A1 at the centre of the observation circle, the receiving antenna no A2 is moved along the surface of the great observation circle. The exploring antenna no A2 is always kept perpendicular to the radius vector and parallel to electric vector. Let I_1 be the current applied to the antenna no A1. Then this current will induce the voltage or e.m.f (E21)in antenna no A2. By rotating the exploring antenna (A2) along the observation circle, the radiation pattern of test antenna (A1) can be obtained. Similarly when the current I_2 applied to antenna A2 will induce voltage E_{12} in the antenna no A1. But $I_1=I_2$, then $E_{12}=E_{21}$. Once the voltages are equal, the power or field strength are equal. Therefore the power pattern or field pattern are equal irrespective of either transmitting or receiving mode.

Similarly the remaining applications can be proved.

18. <u>BEAM WIDTH:</u>

In case of antennas regarding the beam width we need to define two parameters such as Half Power Beam Width (HPBW) and Beam Width between First Nulls (BWFN). These two parameters are represented in the figure 1.13.

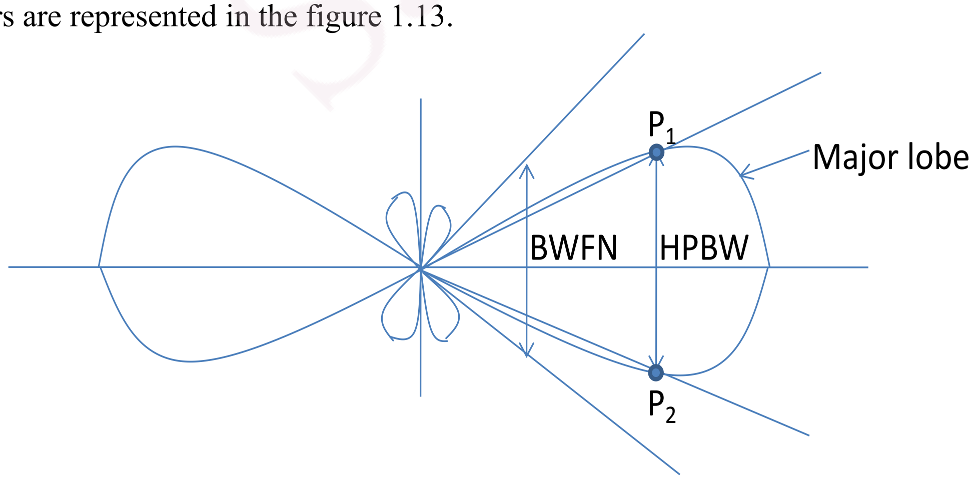


Fig 1.13: Beam Width of an antenna

The antenna beam width is the angular width in degrees measured on the main beam. The half power beam width is defined as the width of the major lobe in between the two half power points P_1 and P_2 as shown in the figure. The half power points are the points at which the power is half of the maximum value. Also the HPBW is defined as the width of the main lobe at which the electric field strength is 0.707 times or 1/√2 times the maximum value. The BWFN is defined as the width of the major lobe in between the first nulls. The BWFN is approximately twice the HPBW. The directivity (D) is related with beam solid angle (dΩ) or beam area (B) as

$$D = \frac{4\pi}{d\Omega} = \frac{4\pi}{B}$$

The beam width place very important role in finding the direction of transmitting antenna. Especially the narrow beam is used in direction finding applications.

19. ANTENNA BEAM EFFICIENCY:

The antenna beam efficiency is defined as the ratio of main beam area (Ω_M) to the total beam area (Ω_A) i.e.

$$\text{Beam Efficiency(BE)} = \frac{\text{Main beam area}}{\text{Total beam area}} = \frac{\Omega_M}{\Omega_A}$$

$$\text{The total beam area}(\Omega_A) = \Omega_M + \Omega_m$$

Where Ω_m is called the minor lobe area.

$$\Omega_A = \Omega_M + \Omega_m$$

Divide on both sides with Ω_A , Then

$$\frac{\Omega_A}{\Omega_A} = \frac{\Omega_M + \Omega_m}{\Omega_A}$$

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$1 = \text{BE} + \text{Stray factor}$$

Therefore Beam Efficiency (BE) = 1-Stray factor.

To have the higher antenna beam efficiency always the stray factor must be as low as possible.

20. BEAM AREA:

Beam area is nothing but a beam solid angle. The solid angle along the radius direction is given by

$$d\Omega = \frac{ds}{r^2} = \frac{r^2 \sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset$$

The beam area or beam solid angle of an antenna is given by the integral of the normalized power pattern over a sphere (4π Steradians)

$$\Omega_A = \iint P_n(\theta, \emptyset) d\Omega$$

The beam area of an antenna can be often be described approximately in terms of the angles subtended by the half power points of the main beam in the two principal planes. Thus,

$$\text{Beam Area} \cong \Omega_A \cong \theta_{HP}\emptyset_{HP}$$

Where θ_{HP} and $\emptyset_{HP}$ are the half power beam widths (HPBW) in the two principal planes, minor lobes being neglected.

21. ANTENNA TEMPERATURE:

The noise power per unit bandwidth at the terminals of a resistor R (shown in figure 1.14) at a temperature T_r is given by

$$p = kT_r \quad (\text{WHz}^{-1})$$

Where

p = power per unit bandwidth in WHz^{-1}

k = Boltzmann's constant = 1.38 x 10^{-23} J K^{-1}

T_r = absolute temperature in Kelvins

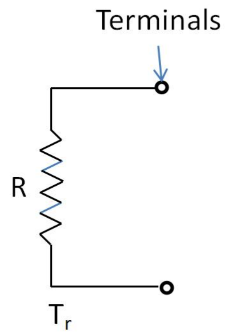


Fig 1.14(a): Resistor at Temparature T_r

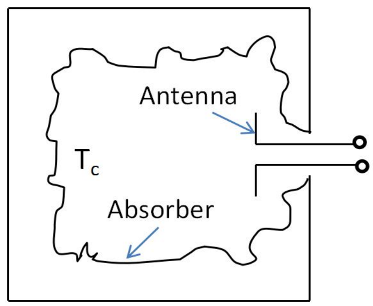


Fig 1.14(b): Antenna in an anechoic Chamber at Temparature T_c

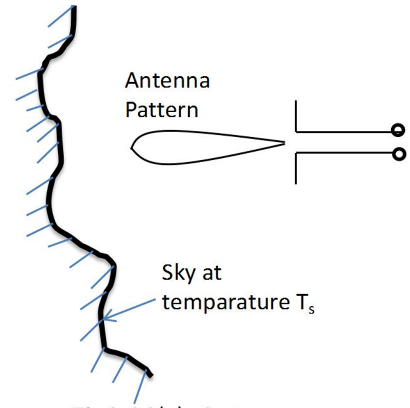


Fig1.14(c): Antenna Observing sky at Temparaturte T_s

When the resistor is replaced with a lossless antenna of radiation resistance R ia anechoic chamber at temperature T_c, then the noise power per bandwidth available at the antenna terminals is same as the above equation and is given by

$$p = kT_c \quad (\text{WHz}^{-1})$$

Where T_c is the temperature of the anehoic chamber

Now if the antenna is removed from the anechoic chamber and pointed at the sky of temperature T_s, the noise power per unit bandwidth is again the same and is given by

$$p = kT_s \quad (\text{WHz}^{-1})$$

Where T_s is the temperature of the sky or atmosphere. Therefore from the above discussion we can say that the sky temperature is nothing but the antenna temperature. The noise power per unit bandwidth in terms of antenna temperature (T_A) is given by

$$p = kT_A \quad (\text{WHz}^{-1})$$

In the above equation it is assumed that the thermal losses of antenna are neglected. Multiplying the above equation with bandwidth (B) we obtain the total available noise power. That is

$$P = kT_A B \quad (\text{Watts})$$

Let us define the flux density of the p, that is

$$S = \frac{p}{A_e} = \frac{kT_A}{A_e} \quad W\, m^{-2} H_z^{-1}$$

Therefore the antenna temperature is given by

$$T_A = \frac{SA_e}{k} \quad in\ degrees\ kelvin$$

22. <u>ANTENNA IMPEDANCE(INPUT IMPEDANCE):</u>

An antenna is a device that integrates the electric field(E) and magnetic field(H) to produce a voltage (V) and current (I) required for actuating an electrical device. An antenna impedance of the antenna is defined as the impedance at the terminals of the antenna where signal is applied. Any antenna can have self impedance and mutual impedance. There are two methods for measurement of antenna impedance such as (i) The poynting vector method and (ii) Induced emf method.

In the poynting vector method, a certain current distribution is assumed to exist along the antenna. The electric and magnetic field strengths due to this assumed current distribution are computed at a point on some surface encircling the antenna.

In the induced emf method, a filamentary current flow is assumed and the resulting electric and magnetic field strengths are computed. Then using reciprocity, the voltages at the antenna terminals produced by all emf's induced along the length of the antenna is computed. The product of this terminal voltage and the in-phase terminal current represents the power radiated.

Once the power flow is computed by either of the above two methods, the self and mutual impedances can be obtained by using conventional relations.

23. <u>POLARIZATION:</u>

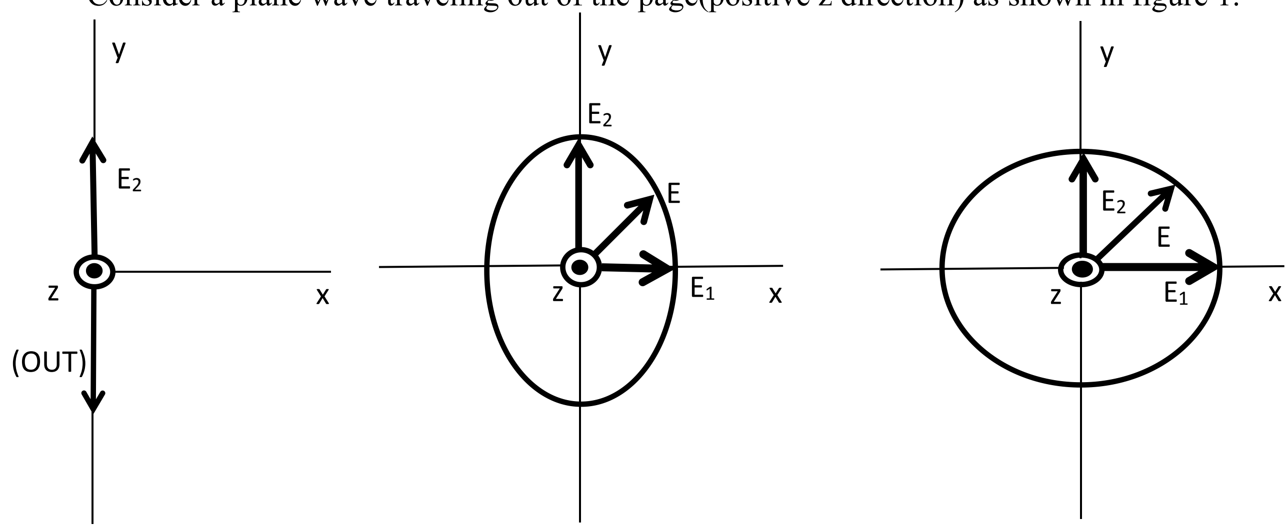
Polarization is defined as the time varying behaviour of the electric filed strength vector at some fixed point in the free space. There are three types of polarizations such as linear polarization, elliptical polarization and circular polarization.

When the two components of electric field having equal or unequal amplitudes and same phase, then the resultant polarization is said to be linear polarization. There are two types of linear polarizations such as linear horizontal polarization and linear vertical polarization. In linear horizontal polarization, the direction of electric vector (E) will be in horizontal direction. Whereas in linear vertical polarization, the direction of vector (E) will be in vertical direction.

When the two components of electric field having unequal amplitudes and any phase difference other than zero, then the resultant electric vector E will trace an elliptical path and hence the polarization is said to be elliptical polarization.

When the two components of electric field having equal amplitudes and 90^0 phase difference, then the resultant electric vector E will trace a circular path and hence the polarization is said to be circular polarization. There are two types of circular polarizations such as right circular polarization and left circular polarization. When phase difference is -90^0, then the resultant electric vector E will rotate in right hand side direction or clock wise direction and hence the polarization is said to be right circular polarization. When phase difference is 90^0, then the resultant electric vector E will rotate in left hand side direction or anti-clock wise direction and hence the polarization is said to be left circular polarization.

Consider a plane wave traveling out of the page(positive z direction) as shown in figure 1.



AR = ∞ | AR = 1.8 | AR = 1

Fig 1(a) Linear polarization | Fig 1(b) Elliptical polarization | Fig 1(c) Circular polarization with the electric field at all times in the y direction. This wave is said to be linearly polarized (in the y direction). As a function of time and position, the electric field is given by

$$E_y = E_2 \sin(\omega t - \beta z) \qquad -(1)$$

In general, the electric field of a wave traveling in the z direction may have both a y component and an x component, as suggested in figure 1(b). In this more general situation, with a phase difference δ between the components, the wave is said to be elliptically polarized. At a fixed value of z the electric vector E rotates as a function of time, the tip of the vector describing an ellipse called the polarization ellipse. The ratio of the major to minor axes of the polarization ellipse is called the Axial Ratio(AR). Thus, for the wave of figure 1(b), AR = E_2/E_1. Two extreme cases of elliptical polarization correspond to circular polarization, as in figure 1(c), and linear polarization, as in figure 1(a). For circular polarization $E_1 = E_2$ and AR = 1, while for linear polarization, $E_1 = 0$ and AR = ∞.

In the most general case of elliptical polarization, the polarization ellipse may have any orientation, as suggested in figure 2. The elliptically polarized wave may be expressed in terms of two linearly polarized components, one in the x direction and one in the y direction. Thus, if the wave is traveling in the positive z direction(out of the page), the electric field components in the x and y directions are

$$E_x = E_1 \sin(\omega t - \beta z) \qquad -(2)$$

$$E_y = E_2 \sin(\omega t - \beta z + \delta) \qquad -(3)$$

Where

E_1 = amplitude of wave linearly polarized in x direction

E_2 = amplitude of wave linearly polarized in y direction

δ = time-phase angle by which E_y leads E_x.

Combining the equations 2 and 3 gives the instantaneous total vector field E:

$$\mathrm{E} = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \sin(\omega t - \beta z + \delta)\, a_y \qquad -(4)$$

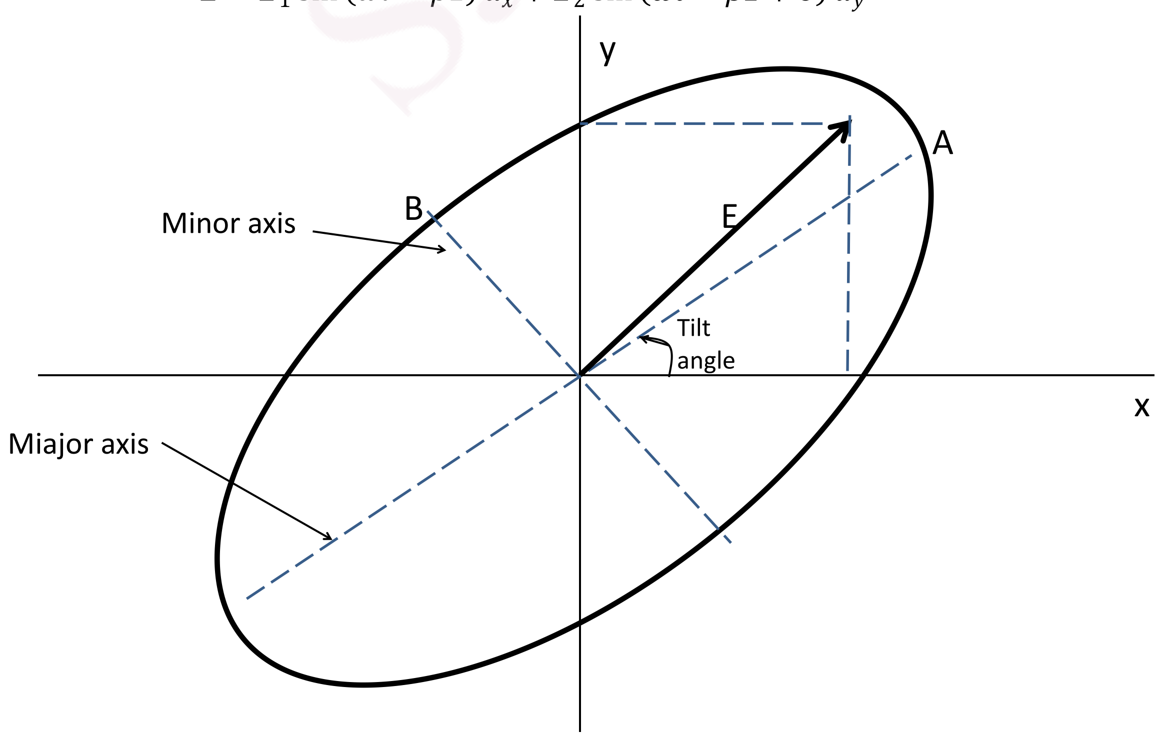


Figure 2: Polarization ellipse

24. BANDWIDTH:

The bandwidth of an antenna is defined as "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard." The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency.

For broadband antennas, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.

For narrowband antennas, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency range of acceptable operation is 5% of the bandwidth center frequency.

25. Friis Transmission Formula(Fundamental Equation for Free Space Propagation):

The Friis transmission formula is derived as follows:

The gain of the antenna under transmitting mode is defined as

$$G_T = \frac{Power\ density\ from\ the\ transmitting\ antenna}{Power\ density\ from\ the\ reference\ antenna} = \frac{P_D}{P_T/_{4\pi R^2}}$$

Where P_T is the power transmitted, R is the distance between the two antennas

$$Power\ density\ transmitted(P_D) = \frac{P_T G_T}{4\pi R^2}$$

When A_e be the effective area of the receiving antenna, then the received power (P_R) is given by

$$P_R = P_D \times A_e = \frac{P_T G_T}{4\pi R^2} \times A_e \qquad -1$$

But the relation between the gain (G_R) of the receiving antenna and its effective area (A_e) is

$$G_R = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{G_R \lambda^2}{4\pi} \qquad -2$$

Substitute equation 2 in equation 1

$$P_R = \frac{P_T G_T}{4\pi R^2} \times \frac{G_R \lambda^2}{4\pi}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

The above equation represents the fundamental equation for the free space propagation.

SOLVED PROBLEMS

1.Find out the directivity or gain of short dipole or oscillating electric dipole

Ans: The directivity or directive gain is defined as

$$\mathrm{D = G = \frac{Power\ density\ from\ the\ test\ anteena(Short\ dipole)}{Power\ density\ from\ the\ reference\ antenna(Isotropic\ radiator)}}$$

$$D = \frac{P_r}{\frac{W_r}{4\pi r^2}} = 4\pi r^2 \frac{P_r}{W_r} \qquad \text{------} \qquad 1$$

Where

P_r=Average pointing vector or power density due to short dipole

$\frac{W_r}{4\pi r^2}$ = power density radiated by the isotropic radiator.

r = distance from the transmitting antenna to the receiving point.

We know that,

$$P_r = \frac{1}{2}(E \times H^*)$$

$$P_r = \frac{1}{2}(|E||H|) \qquad \text{-----} \qquad 2$$

But

$$\frac{E}{H} = \eta \quad or\ E = \eta H \qquad \text{----} \qquad 3$$

Substitute equation 3 in equation 2

$$P_r = \frac{1}{2}\eta|H|^2 \qquad \text{-----} \qquad 4$$

The magnetic field intensity due to short dipole is given by

$$H = H_\emptyset = \frac{j\beta I_m l \sin\theta e^{j\omega\left(t - \frac{r}{c}\right)}}{4\pi r}$$

$$|H| = \frac{\beta I_m l \sin\theta}{4\pi r}$$

For maximum value of H, θ=90^0

$$|H| = \frac{\beta I_m l}{4\pi r} \qquad \text{---------} \qquad 5$$

Substitute equation 5 in equation 4

$$P_r = \frac{1}{2}\eta\left(\frac{\beta I_m l}{4\pi r}\right)^2 = \frac{1}{2}\eta\frac{\beta^2 I_m^2 l^2}{16\pi^2 r^2}$$

But β=2π/λ

$$P_r = \frac{1}{2}\eta\frac{\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{16\pi^2 r^2}$$

$$P_r = \frac{\eta I_m^2 l^2}{8\lambda^2 r^2} \qquad \text{-------} \quad 6$$

The power radiated is given by

$$W_r = I_{rms}^2 R_r \qquad \text{---------} \quad 7$$

Where R_r is the radiation resistance

But the radiation resistance due to short dipole is given by

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \text{------------} \quad 8$$

Where *l* is the length of the short dipole.

Substitute equation 8 in equation 7

$$W_r = I_{rms}^2 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

But $\quad I_{rms} = \frac{I_m}{\sqrt{2}}$

$$W_r = \left(\frac{I_m}{\sqrt{2}}\right)^2 80\pi^2\left(\frac{l}{\lambda}\right)^2 = \frac{80 I_m^2 \pi^2 l^2}{2\lambda^2} \qquad \text{---} \quad 9$$

Substitute equations 6 and 9 in equation 1, then

$$D = 4\pi r^2 \frac{\eta I_m^2 l^2}{8\lambda^2 r^2} \times \frac{2\lambda^2}{80 I_m^2 \pi^2 l^2}$$

But $\eta = 120\pi$

$$D = \frac{\eta}{80\pi} = \frac{3}{2} = 1.5$$

The directivity in dB is given by

$$D = 10 \log (1.5) = 1.76\ dB$$

Therefore the gain or directivity of short dipole is 1.5 or 1.76 dB.

2. Calculate the maximum effective area of short dipole

Ans: Consider the short dipole shown in the figure 1.20 below. The short dipole is used as the receiving antenna.

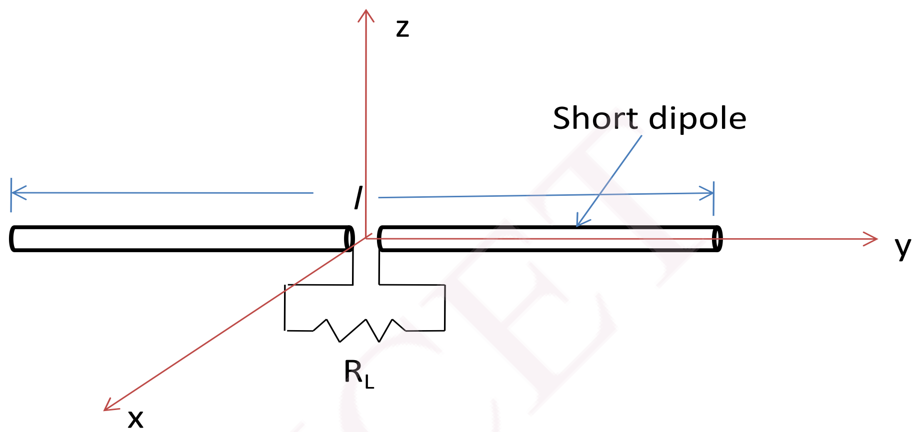


Fig1.20:The short dipole with uniform current along its entire Length and terminated by load resistance R_L

We know that the maximum effective area is

$$A_{em} = \frac{V^2}{4PR_r} \qquad \text{-----1}$$

The induced voltage (V) in a short dipole due to incident electric field (E) is given by

$$V = E \cdot l \qquad \text{----- 2}$$

In above equation it is assumed that, the incident electric field is uniform about the entire length of short dipole.

The pointing vector (P) is given by

$$P = E \times H = \frac{E^2}{\eta} \qquad \text{---- 3}$$

Also the radiation resistance due to short dipole is given b y

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \qquad \text{---- 4}$$

Substitute equations 2,3&4 in equation 1

$$A_{em} = \frac{(E \cdot l)^2}{4\frac{E^2}{\eta} 80\pi^2 \left(\frac{l}{\lambda}\right)^2}$$

$$A_{em} = \frac{E^2 l^2 \eta \lambda^2}{4E^2 80\pi^2 l^2}$$

$$A_{em} = \frac{\eta\lambda^2}{4 \times 80\pi^2} = \frac{120\pi\lambda^2}{4 \times 80\pi^2} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

$$\therefore A_{em} = 0.119\lambda^2$$

3. Calculate the directivity or gain of the half wave dipole

Ans: The half wave dipole is the antenna having the physical length of λ/2.

The directivity or directive gain is defined as

$$\mathrm{D} = \mathrm{G} = \frac{\text{Power density from the test anteena(half wave dipole)}}{\text{Power density from the reference antenna(Isotropic radiator)}}$$

$$D = \frac{P_r}{\frac{W_r}{4\pi r^2}} = 4\pi r^2 \frac{P_r}{W_r} \qquad \text{------} \qquad 1$$

Where

P_r=Average pointing vector or power density due to short dipole

$\frac{W_r}{4\pi r^2}$ = power density radiated by the isotropic radiator.

r = distance from the transmitting antenna to the receiving point.

We know that,

$$P_r = \frac{1}{2}(E \times H^*)$$

$$P_r = \frac{1}{2}(|E||H|) \qquad \text{-----} \qquad 2$$

But

$$\frac{E}{H} = \eta \quad or\ E = \eta H \qquad \text{----} \qquad 3$$

Substitute equation 3 in equation 2

$$P_r = \frac{1}{2}\eta|H|^2 \qquad \text{-----} \qquad 4$$

The magnitude of magnetic field intensity due to half wave dipole is given by

$$|H| = \frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}$$

To have the maximum value for H, take θ=90^0, then above equation becomes

$$|H| = \frac{I_m}{2\pi r} \qquad \text{--}\ 5$$

Substitute equation 5 in equation4

$$P_r = \frac{1}{2}\eta\left(\frac{I_m}{2\pi r}\right)^2 = \frac{\eta I_m^2}{2(4\pi^2 r^2)}$$

$$P_r = \frac{\eta I_m^2}{8\pi^2 r^2} \qquad \text{--} \quad 6$$

The power radiated is given by

$$W_r = I_{rms}^2 R_r \qquad \text{---------} \quad 7$$

Where R_r is the radiation resistance

But the radiation resistance due to half wave dipole is given by

$$R_r = 73\Omega \qquad \text{---} \quad 8$$

Substitute equation 8 in equation 7, then

$$W_r = I_{rms}^2(73)$$

But $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$W_r = \left(\frac{I_m}{\sqrt{2}}\right)^2 (73)$$

$$W_r = \frac{73 I_m^2}{2} \quad \text{--} \quad 9$$

Substitute equations 6 and 9 in equation1

$$D = 4\pi r^2 \frac{\frac{\eta I_m^2}{8\pi^2 r^2}}{\frac{73 I_m^2}{2}} = 4\pi r^2 \frac{\eta I_m^2}{8\pi^2 r^2} \times \frac{2}{73 I_m^2}$$

$$D = \frac{\eta}{73\pi}$$

But $\eta = 120\pi$

$$D = \frac{120\pi}{73\pi} = \frac{120}{73} = 1.64$$

The directivity in dB is given by

$$D = 10 \log(1.64) = 2.148\, dB$$

Therefore the gain or directivity of half wave dipole is 1.64 or 2.148 dB.

4. Calculate the maximum effective area of half wave dipole

We know that the maximum effective area is

$$A_{em} = \frac{V^2}{4PR_r} \quad \text{-----}1$$

To find out the voltage induced in the half wave dipole due to incident electric field we need to consider the current distribution on the half wave dipole which is shown in the figure1.21 below

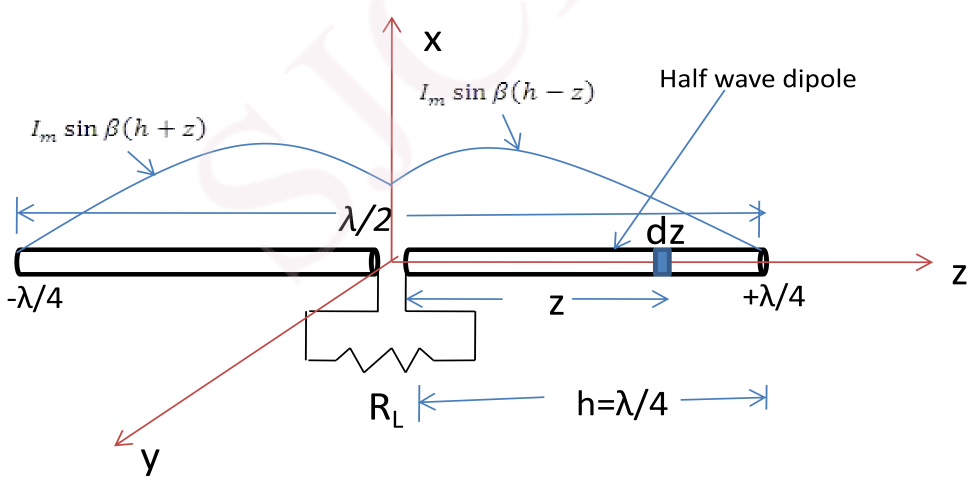


Fig1.21: Half wave dipole and its current distribution

$$I = I_m \sin\beta(h-z) \quad for \quad z > 0$$
$$I = I_m \sin\beta(h+z) \quad for \quad z < 0$$

But h = λ/4

$$I = I_m \sin\beta(h-z) = I_m \sin\beta\left(\frac{\lambda}{4} - z\right) = I_m \sin\left(\beta\left(\frac{\lambda}{4}\right) - \beta z\right)$$

$$I = I_m \sin\left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) - \beta z\right) = I_m \sin\left(\frac{\pi}{2} - \beta z\right)$$

$$I = I_m \cos \beta z \qquad -2$$

The above equation can also written as

$$dI = dI_m \cos \beta z$$

Or

$$dV = dV_m \cos \beta z \qquad -3$$

We know that,

$$V = E \cdot l$$

Or

$$dV_m = E \cdot dl = E \cdot dz \qquad -4$$

Substitute equation 4 in equation 3

$$dV = E \cdot dz \cos \beta z$$
$$dV = E\, cos\beta z\, dz$$

Take integration on both sides

$$\int dV = \int E\, cos\beta z\, dz$$

$$V = \int_{-\lambda/4}^{\lambda/4} E\, cos\beta z\, dz$$

$$V = 2\int_{0}^{\lambda/4} E\, cos\beta z\, dz$$

$$V = 2E\left[\frac{sin\beta z}{\beta}\right]_0^{\lambda/4}$$

$$V = \frac{2E}{\beta}(\sin \beta(\lambda/4) - \sin \beta(0))$$

$$V = \frac{2E}{\frac{2\pi}{\lambda}}\left(\sin \frac{2\pi}{\lambda}\frac{\lambda}{4} - 0\right)$$

$$V = \frac{E\lambda}{\pi} \qquad -5$$

$$P = E \times H = \frac{E^2}{\eta} \qquad -6$$

Substitute equation 5 and 6 in equation 1

$$A_{em} = \frac{\left(\frac{E\lambda}{\pi}\right)^2}{4\frac{E^2}{\eta}R_r} \qquad -7$$

The radiation resistance due to half wave dipole is given by

$$R_r = 73\, \Omega \qquad -8$$

Substitute equation 8 in equation 7

$$A_{em} = \frac{\left(\frac{E\lambda}{\pi}\right)^2}{4\frac{E^2}{\eta}(73)} = \frac{E^2\lambda^2\eta}{4E^2\pi^2(73)} = \frac{\lambda^2\eta}{4\pi^2(73)}$$

But $\eta = 120\,\pi$

$$A_{em} = \frac{\lambda^2 120\pi}{4\pi^2(73)} = 0.13\,\lambda^2$$

5. Calculate the effective length of the half wave dipole

Sol:

The effective length is given by

$$l_e = 2\sqrt{\frac{A_{em}R_r}{\eta}} \qquad -1$$

For half wave dipole

$$\mathrm{R_r} = 73\,\Omega \qquad -2$$

$$\mathrm{A_{em}} = 0.13\,\lambda^2 \qquad -3$$

Substitute equations 2 and 3 in equation 1

$$l_e = 2\sqrt{\frac{0.13\,\lambda^2(73)}{120\pi}} = 0.3174\,\lambda$$

6. Find out the directivity of isotropic radiator

Sol:

The formula for directivity is given by

$$D = \frac{4\pi}{Beam\ area} = \frac{4\pi}{4\pi} = 1$$

7. Calculate the maximum effective aperture of a microwave antenna which has a directivity of 900.

Sol:

The relation between the directivity and effective area is given by

$$D = G = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{D\,\lambda^2}{4\pi} = \frac{900\,\lambda^2}{4\pi} = 71.619\,\lambda^2$$

8. An antenna has a radiation resistance of 72 ohms, a loss resistance of 8 ohms and power gain of 12 dB. Determine the antenna efficiency and its directivity.

Sol:

Given data:

$$R_r = 72\,\Omega,\ R_l = 8\Omega,\ G_p = 12\,dB = 15.85$$

The antenna efficiency factor is given by

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72+8} = 0.9$$

Antenna efficiency = 0.9 X 100 = 90%

The relation between the power gain and directive gain is given by

$$\eta = \frac{G_p}{G_d} = \frac{G_p}{D}$$

$$G_d = D = \frac{G_p}{\eta} = \frac{15.85}{0.9} = 17.611$$

Or

$$D = 10\log(17.611) = 12.458\,dB$$

9. A Low frequency transmitting antenna has a radiation resistance of 0.5 ohms and a total loss of 2.5 ohms. Calculate the radiated power, power input and antenna efficiency if the current applied to the antenna is 100 A(rms).

Sol:

Given data:

$$R_r = 0.5\,\Omega,\ \ R_l = 2.5\Omega,\ \ I_{rms} = 100\ A$$

We know that, the power radiated is given by

$$W_r = I_{rms}^2 R_r = 100^2(0.5) = 5\ kW$$

Power input is given by

$$W_T = I_{rms}^2(R_A) = I_{rms}^2(R_r + R_l) = 100^2(0.5 + 2.5) = 30\ kW$$

The antenna efficiency is given by

$$\eta = \frac{R_r}{R_r + R_l} \times 100 = \frac{0.5}{0.5 + 2.5} \times 100 = 16.6\%$$

10. An antenna has a field pattern given by

$$E(\theta) = \cos^2\theta, \quad \textbf{for}\ \ 0^0 \le \theta \le 90^0$$

Find Half Power Beamwidth(HPBW)

Sol:

$$E(\theta) = \cos^2\theta, \quad \text{for}\ \ 0^0 \le \theta \le 90^0$$

At half power points, the electric field will be

$$E(\theta) = 0.707$$

$$0.707 = \cos^2\theta$$

$$\cos\theta = \sqrt{0.707}$$

$$\theta = \cos^{-1}\left(\sqrt{0.707}\right) = 33^0$$

$$HPBW = 2\times\theta = 2\times 33 = 66^0$$

11. An antenna has a field pattern given by

$$E(\theta) = \cos\theta\cos 2\theta, \quad \textbf{for}\ \ 0^0 \le \theta \le 90^0$$

Find (a) Half Power Beamwidth(HPBW) and (b) Beamwidth between First Nulls(FNBW)

Sol:

$$E(\theta) = \cos\theta\cos 2\theta, \quad \textbf{for}\ \ 0^0 \le \theta \le 90^0$$

At half power points, the electric field will be

$$E(\theta) = 0.707 = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \cos\theta\cos 2\theta$$

$$\cos 2\theta = \frac{1}{\sqrt{2}\cos\theta}$$

$$2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta'}\right)$$

By iterating with $\theta' = 0$ as a first guess,

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(0)}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 22.5^0$$

Let $\theta' = 22.5^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(22.5)}\right) = 20.03^0$$

Let $\theta' = 20.03^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.03)}\right) = 20.59^0$$

Let $\theta' = 20.59^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.59)}\right) = 20.47^0$$

Let $\theta' = 20.47^0$, then

$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos(20.47)}\right) = 20.47^0 \cong 20.5^0$$

Therefore,

$$\theta = \theta' = 20.5^0$$

(a) The Half Power Beamwidth is given by

$$HPBW = 2\times\theta = 2\times 20.5 = 41^0$$

(b) The First Null Beam Width (FNBW) is obtained as follows:
At FNBW, the electric filed will be zero.

$$E(\theta) = 0$$

$$0 = \cos\theta\cos 2\theta$$

$$\cos 2\theta = 0$$

$$2\theta = \cos^{-1}(0) = 90^0$$

$$\theta = \frac{90}{2} = 45^0$$

$$FNBW = 2\times\theta = 2\times 45 = 90^0$$

12. Find the number of square degrees in the solid angle Ω on a spherical surface that is between θ = 20⁰ and θ = 40⁰ and between ϕ = 30⁰ and ϕ = 70⁰.

Sol:

We know that,

$$ds = r^2 \sin\theta \, d\theta \, d\emptyset$$

Solid angle is given by

$$d\Omega = \frac{ds}{r^2}$$

$$d\Omega = \frac{r^2 \sin\theta \, d\theta \, d\emptyset}{r^2} = \sin\theta \, d\theta \, d\emptyset$$

$$d\Omega = \sin\theta \, d\theta \, d\emptyset$$

Take integration on both sides

$$\Omega = \int \sin\theta \, d\theta \, d\emptyset$$

$$\Omega = \int_{20}^{40} \sin\theta \, d\theta \int_{30}^{70} d\emptyset$$

$$\Omega = [-\cos\theta]_{20}^{40} \times [\emptyset]_{30}^{70}$$

$$\Omega = (-\cos 40 + \cos 20) \times (70 - 30) \times \frac{\pi}{180} = 0.121 \text{ steradians}$$

$$\Omega = 0.121 \times \left(\frac{180}{\pi}\right)^2 = 397 \text{ square degrees}$$

13. An antenna has a field pattern given by

$$E(\theta) = \cos^2\theta, \quad \textbf{for} \quad 0^0 \le \theta \le 90^0$$

Find the beam area of this pattern

Sol:

$$E(\theta) = \cos^2\theta, \quad \textbf{for} \quad 0^0 \le \theta \le 90^0$$

At half power points, the electric field will be

$$E(\theta) = 0.707$$

$$0.707 = \cos^2\theta$$

$$\cos\theta = \sqrt{0.707}$$

$$\theta = \cos^{-1}\left(\sqrt{0.707}\right) = 33^0$$

$$HPBW = 2 \times \theta = 2 \times 33 = 66^0$$

$$\theta_{HP} = \emptyset_{HP} = 66^0$$

The approximate formula for beam area is given by

$$\Omega_A = \theta_{HP}\emptyset_{HP}$$

$$\Omega_A = 66 \times 66 = 4356 \text{ Square degrees}$$

But one square radian is equal to 3283 square degrees.

$$\Omega_A = \frac{4356}{3283} = 1.33 \text{ steradian}$$

14. A radio link has a 15-W transmitter connected to an antenna of 2.5 m^2 effective aperture at 5 GHz. The receiving antenna has an effective aperture of 0.5 m^2 and is located at a 15-km line-of-sight distance from the transmitting antenna. Assume lossless, matched antennas, find the power delivered to the receiver.

Sol:

Transmitted power (P_T) = 15 W

Effective aperture of transmitting antenna (A_{eT}) = 2.5 m^2

Frequency of operation (f) = 5 GHz

Wavelength (λ) = c/f = $(3X10^8)/(5X10^9)$ = 0.06

Effective aperture of receiving antenna (A_{eR}) = 0.5 m^2

Distance between transmitter and receiver (R) = 15 km

$$G_T = \frac{4\pi A_{eT}}{\lambda^2}$$

$$G_R = \frac{4\pi A_{eR}}{\lambda^2}$$

The power received by receiver is given by

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_R = P_T \left(\frac{4\pi A_{eT}}{\lambda^2}\right)\left(\frac{4\pi A_{eR}}{\lambda^2}\right)\left(\frac{\lambda}{4\pi R}\right)^2$$

$$P_R = P_T \frac{A_{eT} A_{eR}}{R^2 \lambda^2}$$

$$P_R = 15 \times \frac{2.5 \times 0.5}{15^2 \times 0.06^2}$$

$$P_R = 23\mu W$$

15. An elliptically polarized wave traveling in the positive z direction in air has x and y components.

$$\boldsymbol{E_x = 3\sin(\omega t - \beta x) \quad V/m}$$
$$\boldsymbol{E_y = 6\sin(\omega t - \beta x + 75^0) \quad V/m}$$

Find the average power per unit area conveyed by the wave.

Sol:

$$E_x = 3\sin(\omega t - \beta x) \quad V/m$$
$$E_y = 6\sin(\omega t - \beta x + 75^0) \quad V/m$$

From poynting theorem, the average poynting vector(Average power per unit area) is given by

$$P_{av} = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2}\frac{E_x^2 + E_y^2}{\eta}$$

$$P_{av} = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2} \times \frac{3^2 + 6^2}{377} = 60\ mW/m^2$$

16. Calculate the physical height of a half wave dipole (λ/2) having antenna Q of 30 and bandwidth of 10 Mhz.

Sol:

Quality factor (Q) = 30
Bandwidth (BW) = 10 MHz

$$Q = \frac{f}{BW}$$

$$f = Q \times BW = 30 \times 10 \times 10^6 = 3 \times 10^8\, Hz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^8} = 1m$$

The physical height of half wave dipole is

$$l = \frac{\lambda}{2} = \frac{1}{2} = 0.5m$$

17. A resonant half wave length dipole is made out of copper (σ = 10X10^7 siemen/m). Calculate the conduction dielectric radiation efficiency of the dipole antenna at f = 100 MHz if the radius of the wire is r_0 = 3 X 10^{-4} λ and radiation resistance of the λ/2 dipole is 73 ohms.

Sol:

Conductivity (σ) = 10X10^7 siemen/m
Frequency (f) = 100 MHz
Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{100\times10^6} = 3m$$

Radius of the wire is(r_0) = 3 X 10^{-4} λ
Radiation resistance (R_r) = 73 ohms
The loss resistance at high frequency is given by

$$R_{hf} = R_l = \frac{l}{2\pi r_0} R_s = \frac{l}{2\pi r_0}\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R_l = \frac{\lambda/2}{2\pi r_0}\sqrt{\frac{2\pi f\mu}{2\sigma}}$$

$$R_l = \frac{3/2}{2\pi\times3\times10^{-4}\times3}\sqrt{\frac{2\pi\times100\times10^6\times4\pi\times10^{-7}}{2\times10\times10^7}} = 0.698\Omega$$

Antenna efficiency is given by

$$\eta = \frac{R_r}{R_r + R_l}\times100 = \frac{73}{73+0.698}\times100 = 99.052\%$$

UNIT-II (Wire and Antenna Arrays)

Power radiated and radiation resistance due to small electric dipole or short dipole or oscillating electric dipole:

The general equation for the power radiated due to any antenna is given by

$$W = \int P_{av} \cdot ds \qquad -1$$

Where P_{av} is the average poynting vector.

From the poynting theorem we have

$$P_{av} = \frac{1}{2}(E \times H) = \frac{1}{2}\eta|H|^2 \qquad -2$$

The magnetic field intensity due to small electric dipole for radiation zone is given by

$$H = H_\varphi = \frac{j\omega l \sin\theta\, I_m\, e^{j\omega\left(t-\frac{r}{c}\right)}}{4\pi c r}$$

$$|H| = \frac{\omega l \sin\theta\, I_m}{4\pi c r} \qquad -3$$

Substitute equation 3 in equation 2

$$P_{av} = \frac{1}{2}\eta\left(\frac{\omega l \sin\theta\, I_m}{4\pi c r}\right)^2 \qquad -4$$

Substitute equation 4 in equation 1

$$W = \int \frac{1}{2}\eta\left(\frac{\omega l \sin\theta\, I_m}{4\pi c r}\right)^2 \cdot ds$$

But $ds = r^2 \sin\theta d\theta d\varphi$

$$W = \int \frac{1}{2}\eta\left(\frac{\omega l \sin\theta\, I_m}{4\pi c r}\right)^2 r^2 \sin\theta d\theta d\varphi$$

$$W = \frac{1}{2}\eta\frac{\omega^2 I_m^2 l^2}{16\pi^2 c^2}\int_0^{2\pi} d\varphi \int_0^{\pi} sin^3\theta d\theta$$

$$W = \frac{1}{2}\eta\frac{\omega^2 I_m^2 l^2}{16\pi^2 c^2}(2\pi)\left(\frac{4}{3}\right)$$

$$W = \eta\frac{\omega^2 I_m^2 l^2}{12\pi c^2} = \eta\frac{\beta^2 I_m^2 l^2}{12\pi}$$

$$W = \frac{\eta(\beta I_m l)^2}{12\pi} \qquad - 5$$

The above equation represents the power radiated by the small electric dipole.

In general the average power applied to the antenna is given by

$$W = \frac{1}{2}I_m^2 R_r \qquad -6$$

Where I_m is the peak value of current applied to the antenna and R_r is the radiation resistance.

Equate equations 5 and 6

$$\frac{1}{2}I_m^2 R_r = \frac{\eta\left((\beta I_m l)\right)^2}{12\pi}$$

$$\frac{1}{2}I_m^2R_r = \frac{\eta\beta^2I_m^2l^2}{12\pi}$$

$$R_r = \frac{\eta\beta^2l^2}{6\pi} = \frac{(120\pi)}{6\pi}\left(\frac{2\pi}{\lambda}\right)^2 l^2$$

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \qquad -7$$

The above equation represents the radiation resistance of small electric dipole.

<u>Power radiated and radiation resistance due to half wave dipole:</u>

The general equation for the power radiated due to any antenna is given by

$$W = \int P_{av} \cdot ds \qquad -1$$

Where P_{av} is the average poynting vector.

From the poynting theorem we have

$$P_{av} = \frac{1}{2}(E \times H) = \frac{1}{2}\eta|H|^2 \qquad -2$$

The magnetic field intensity due to hale wave dipole for radiation zone is given by

$$H = H_\varphi = \frac{jI_m}{2\pi r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}e^{-j\beta r}$$

$$|H| = \frac{I_m}{2\pi r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \qquad -3$$

Substitute equation 3 in equation 2

$$P_{av} = \frac{1}{2}\eta\left(\frac{I_m}{2\pi r}\ \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2$$

$$P_{av} = \frac{1}{2}(120\pi)\frac{I_m^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{4\pi^2r^2sin^2\theta}$$

$$P_{av} = \frac{15I_m^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2sin^2\theta}$$

$$P_{av} = \frac{15\left(\sqrt{2}I_{rms}^2\right)^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2sin^2\theta}$$

$$P_{av} = \frac{30I_{rms}^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2sin^2\theta} \qquad -4$$

Substitute equation 4 in equation1

$$W = \int \frac{30I_{rms}^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2sin^2\theta} \cdot ds$$

But $ds = r^2\sin\theta d\theta d\varphi$

$$W = \int \frac{30I_{rms}^2cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi r^2sin^2\theta} \cdot r^2\sin\theta d\theta d\varphi$$

$$W = \int \frac{30I_{rms}^2 cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\pi \sin\theta} d\theta d\varphi$$

$$W = \frac{30I_{rms}^2}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$W = \frac{30I_{rms}^2}{\pi}(2\pi)\int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$W = 60\ I_{rms}^2 \int_0^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

In above equation the integral part can be evaluated by using either analytical methods or numerical methods such as Simpson's or Trapezoidal method. The value of the integral part is equal to 1.219

$$W = 60\ I_{rms}^2(1.219)$$

$$W = 73\ I_{rms}^2 \qquad -5$$

The above equation represents power radiated due to half wave dipole.

We know that

$$W = I_{rms}^2 R_r \qquad -6$$

By comparing equations 5 and 6 we can say that the radiation resistance is

$$R_r = 73\ \Omega$$

Power radiated and radiation resistance due to monopole or quarter monopole:

Quarter wave monopole is a antenna having the length of λ/4. The radiation resistance due to quarter monopole is equal to half of the radiation resistance of half wave dipole. Therefore, the radiation resistance of monopole is 36.5 Ω.

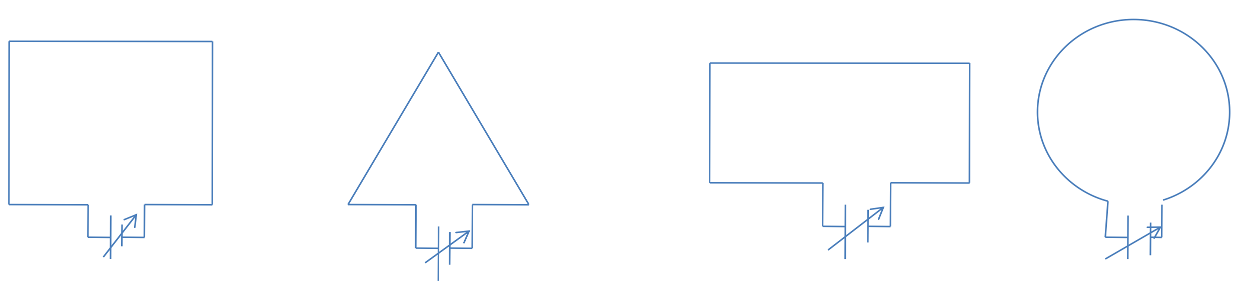
The power radiated due to monopole is given by

$$W = 36.5\ I_{rms}^2$$

LOOP ANTENNAS

Introduction:

Loop antenna is defined as a radiating coil of any convenient cross section of one or more turns carrying RF (Radio Frequency) current. It may assume any shape such as rectangular, square, triangular, hexagonal and circular. The following figure shows the loop antennas of different shapes.



Square loop Triangular loop Rectangular loop Circular loop

Small Loop:

A loop antenna is said to be small if its cross sectional is small. Let us consider the two cases of small loop antenna such as receiving case and transmitting case.

(i) Receiving loop antenna:

To derive the equation for the e.m.f or voltage induced in the receiving loop antenna, let us consider the rectangular loop antenna shown in the figure below.

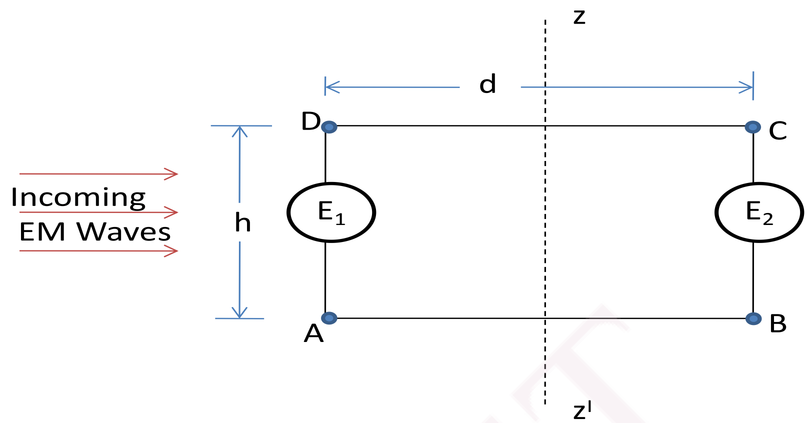
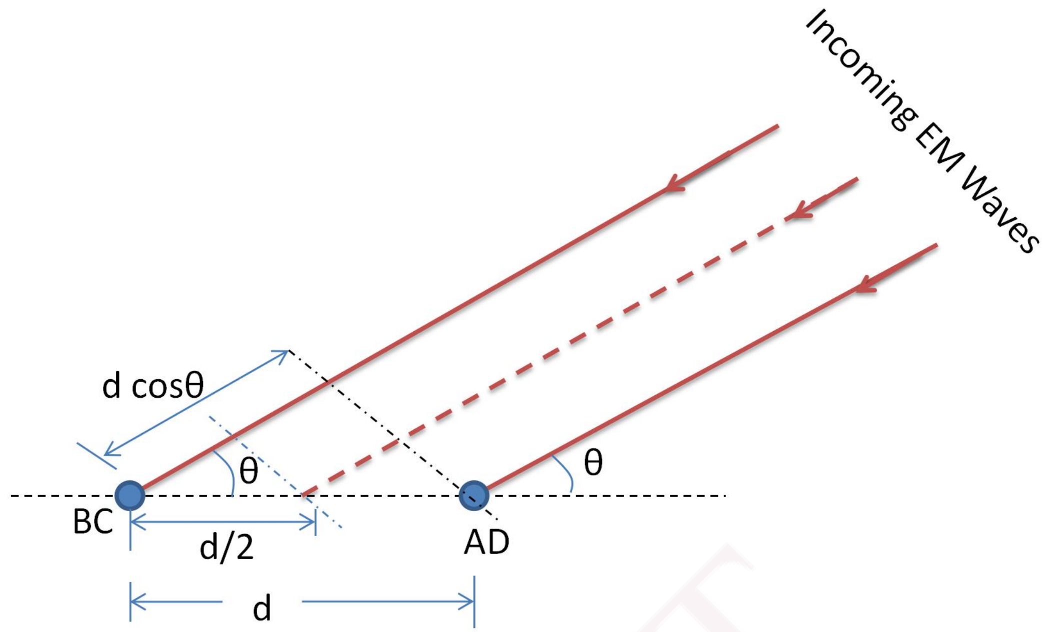


Fig: Loop antenna with rotational axis

For the sake of explanation of let us assume that, the incoming EM waves are vertically polarized. The rectangular loop antenna can be imagined as a combination of two vertical antennas (AD & BC) and two horizontal antennas (AB & CD). When the axis of the plane of the loop is perpendicular with respect to the incoming EM waves, then the two vertical antennas will receive the same amount of signal but the resultant voltage induced is zero because the vertical antennas will be at equidistance with respect to incoming waves. When the axis of the plane of the loop is in parallel with incoming EM waves, then the there will be maximum induced voltage. From the figure shown below we can find the voltage induced in the loop antenna.



The path difference between the waves due to antenna AD w.r.t center or waves due to antenna BC w.r.t center is given by

$$Path\ difference(P.D) = \frac{d}{2}cos\theta \qquad -1$$

The phase angle difference due to path difference is given by

$$\alpha = \frac{2\pi}{\lambda}(P.D) = \frac{2\pi}{\lambda}\left(\frac{d}{2}cos\theta\right) = \frac{\pi d cos\theta}{\lambda} \qquad -2$$

Le the incident electric field at the center of the two antennas is $E_m sin\omega t$, then the voltage induced in the antenna AD is given by

$$V_1 = E_m \sin(\omega t + \alpha).h \qquad -3$$

Similarly the voltage induced in antenna BC is given by

$$V_2 = E_m \sin(\omega t - \alpha).h \qquad -4$$

The resultant voltage induced is given by

$$e = V_1 - V_2 \qquad -5$$

Substitute equations 3 and 4 in 5

$$e = E_m \sin(\omega t + \alpha).h - E_m \sin(\omega t - \alpha).h$$

$$e = E_m h[(\sin(\omega t + \alpha) - \sin(\omega t - \alpha)]$$

$$e = E_m h[sin\omega t.cos\alpha + cos\omega t.sin\alpha - (sin\omega t.cos\alpha + cos\omega t.sin\alpha)]$$

$$e = 2E_m h\ cos\omega t.sin\alpha \qquad -6$$

Now substitute equation 2 in equation 6

$$e = 2E_m h\ cos\omega t.sin\left(\frac{\pi d cos\theta}{\lambda}\right)$$

But when d << λ, then $sin\left(\frac{\pi d cos\theta}{\lambda}\right) = \left(\frac{\pi d cos\theta}{\lambda}\right)$

$$e = 2E_m h\, cos\omega t.\left(\frac{\pi d cos\theta}{\lambda}\right)$$
$$e = \frac{2\pi hd\, cos\theta}{\lambda} E_m cos\omega t$$

$$e = V_m cos\omega t$$

Where V_m called as magnitude of the induced voltage and is given by

$$V_m = \frac{2\pi hd\, cos\theta}{\lambda} E_m = \frac{2\pi AN\, cos\theta}{\lambda} E_m$$

Where A = hd is known as area of the loop and N is the no.of turns of the loop antenna.

(ii) Transmitting loop Antenna:

To derive the far field components of the loop antenna under the transmitting mode, let us consider the square loop located at the center of the spherical coordinate system as shown in the figure below.

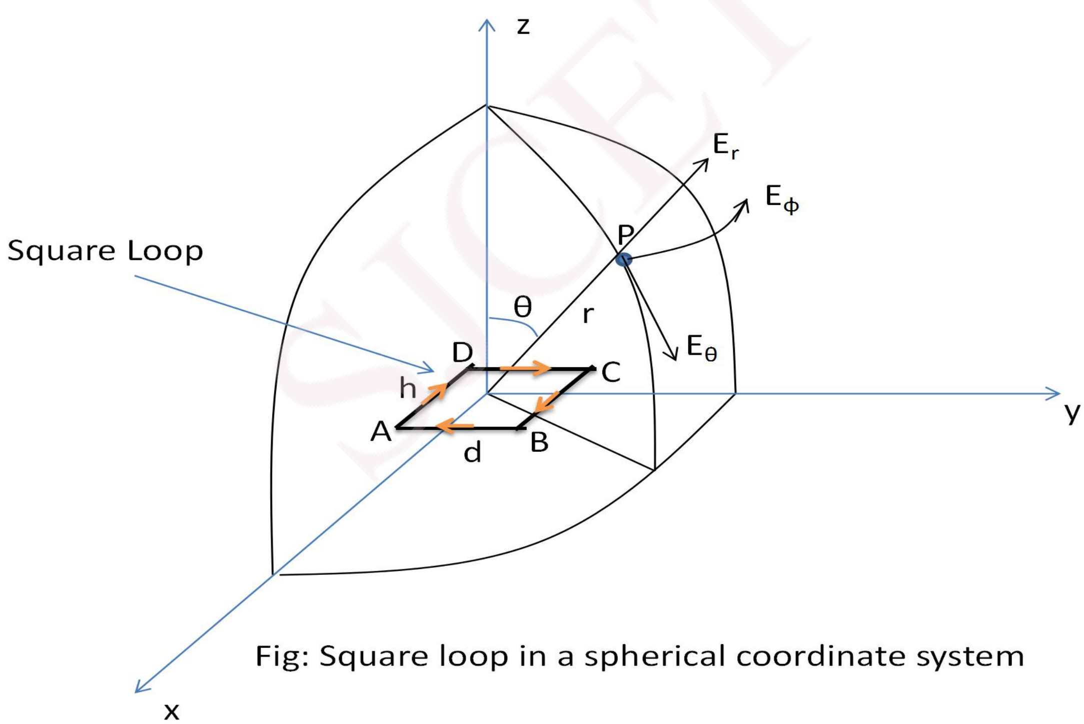
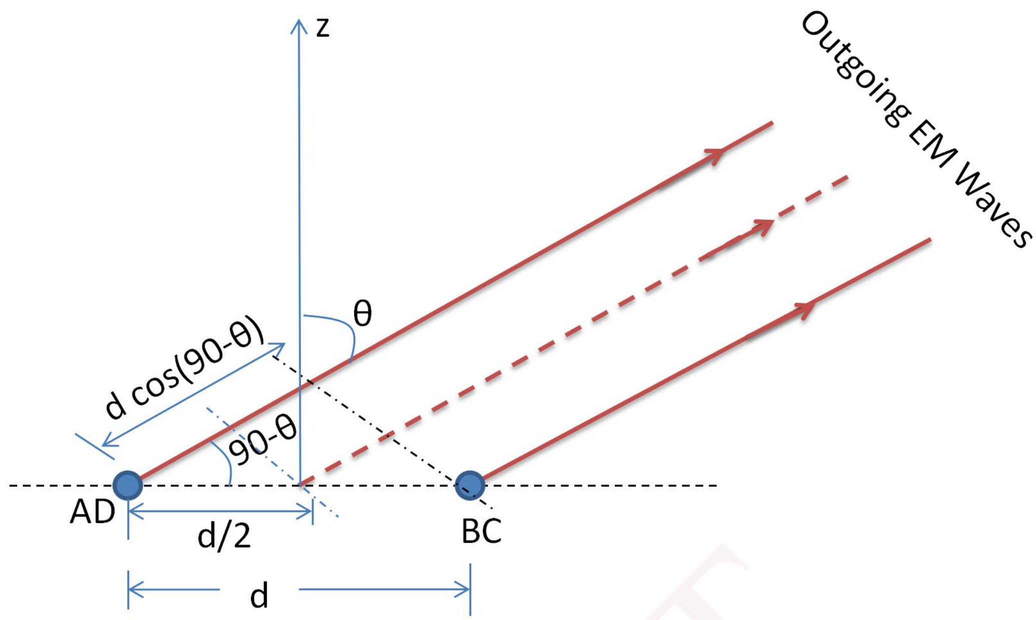


Fig: Square loop in a spherical coordinate system



From the second figure shown in above, we can derive the field components of the loop antenna. The path difference between the waves due to two antennas AD and BC is given by

$$Path\ difference(P.D) = dcos(90^0 - \theta) = d\ sin\theta \qquad -1$$

$$\Psi = \frac{2\pi}{\lambda}(P.D) = \frac{2\pi}{\lambda}(d\ sin\theta) = \beta\ dsin\theta \qquad -2$$

The total electric field at the receiving point due two antennas AD and BC is given by

$$E_\emptyset = E_0 e^{-j\Psi/2} - E_0 e^{j\Psi/2} =- E_0\left(e^{j\Psi/2} - e^{-j\Psi/2}\right)$$

$$E_\emptyset =- \frac{2j}{2j} E_0\left(e^{j\Psi/2} - e^{-j\Psi/2}\right) =- 2jE_0\left(\frac{e^{j\Psi/2} - e^{-j\Psi/2}}{2j}\right) =- 2jE_0 sin(\Psi/2) \quad -3$$

Substitute equation 2 in equation 3

$$E_\emptyset =- 2jE_0 sin\left(\frac{\beta\ dsin\theta}{2}\right) =- 2jE_0\left(\frac{\beta\ dsin\theta}{2}\right) \quad when\ d \ll \lambda$$

$$E_\emptyset =- jE_0\beta d\ sin\theta \qquad -4$$

In above equation E_0 is known as individual field component which can be obtained from the short dipole.

We know that, the far field component of short dipole is

$$E_\theta = E_0 = \frac{j60\pi[I]h}{r\lambda} \qquad -5$$

Substitute equation 5 in equation 4

$$E_\emptyset = -j\left(\frac{j60\pi[I]h}{r\lambda}\right)\beta d\, sin\theta = \frac{120\pi^2[I]A\, sin\theta}{r\lambda^2} \qquad -6$$

We know that

$$\frac{E}{H} = \eta$$

$$\frac{E_\emptyset}{H_\theta} = \eta$$

Then,

$$H_\theta = \frac{E_\emptyset}{\eta} = \frac{E_\emptyset}{120\pi} \qquad -7$$

Substitute equation 6 in equation 7

$$H_\theta = \frac{E_\emptyset}{120\pi} = \frac{\frac{120\pi^2[I]A\, sin\theta}{r\lambda^2}}{120\pi} = \frac{\pi[I]A\, sin\theta}{r\lambda^2}$$

Comparison of far fields of small loop and short dipole:

The following table gives the comparison of far fields of small loop and short dipole. From the table it can be observed the following points.

(i) The field components of short dipole includes the parameter j indicates that, the field components due to short dipole are in time phase quadrature as compared with the filed components due to loop antenna.

(ii) The field components due to loop antenna are inversely proportional to the square of the wave length λ where as the field components due to short dipole are inversely proportional to wave length λ

Field	Short dipole	Loop antenna
Electric filed	$E_\theta = \frac{j60\pi[I]l\, sin\theta}{r\lambda}$	$E_\emptyset = \frac{120\pi^2[I]A\, sin\theta}{r\lambda^2}$
Magnetic filed	$\boldsymbol{H}_\emptyset = \frac{j[I]l\, sin\theta}{2r\lambda}$	$H_\theta = \frac{\pi[I]A\, sin\theta}{r\lambda^2}$

Radiation Resistances and Directives of small and large loops (Qualitative Treatment):

The radiation resistance of loop antenna is given by

$$R_r = 31{,}200\left(\frac{NA}{\lambda^2}\right)^2 \quad when\ the\ loop\ is\ small$$

$$R_r = 592\, C_\lambda \quad when\ the\ loop\ is\ large$$

Where C_λ is known as circumference in wavelength. For circular loop $C_\lambda = 2\pi a/\lambda$ where 'a' is the radius of the loop.

The directivity of the loop antenna is given by

$$Directiviyt(D) = \frac{3}{2} \quad when\ the\ loop\ is\ small\ i.e\ when\ C_\lambda < 1/3$$

$$Directiviyt(D) = 0.68\, C_\lambda \quad when\ the\ loop\ is\ small\ i.e\ when\ C_\lambda > 2$$

Applications of Loop Antennas:

The following are the list of applications of loop antennas

(i) In direction finding applications
(ii) Radio receivers
(iii) UHF transmitters
(iv) Aircraft receivers

ARRAYS OF 2 ISOTROPIC SOURCES- DIFFERENT CASES

Introduction:

Array is defined as method of combining the radiations from the group or array of elements (antenna) with involving wave interference. The total field at a distance point 'P' due to the antenna array is the vector sum of the fields produced by the individual antennas of the array system. Array is said to be linear, when all the elements are equally spaced along straight line. Further a array is said to be uniform linear array, if all the elements in the array are fed with currents of equal amplitudes and uniform progressive phase shift along the line. There are different types of antennas arrays such as Broad side array, End fire array, Collinear array and parasitic array. The structure of 7 element broad side and end fire array is shown in figure below.

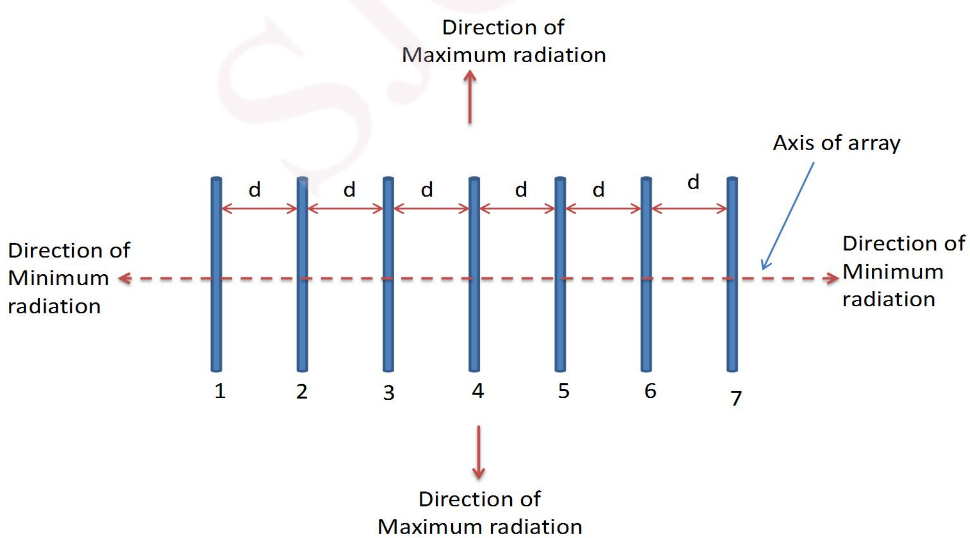


Fig: Broad side array arrangement

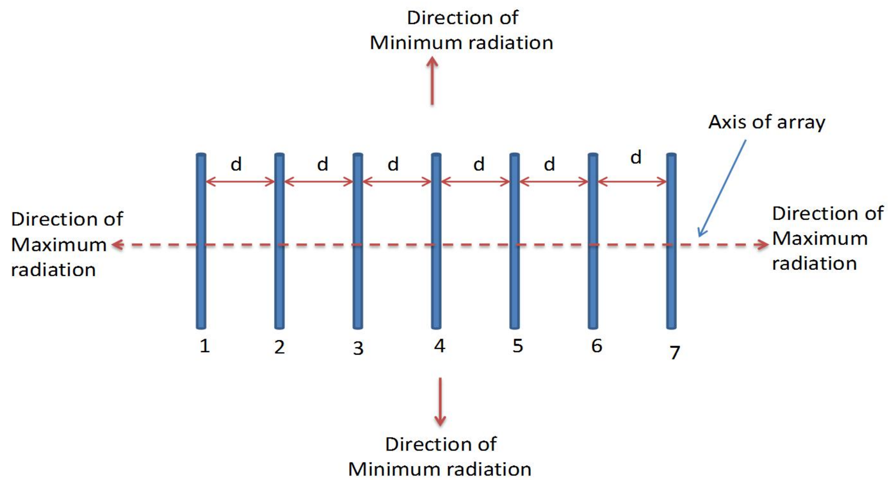
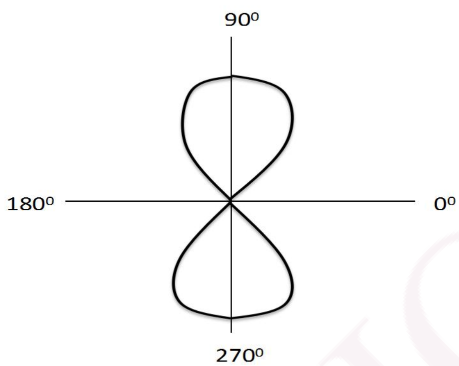
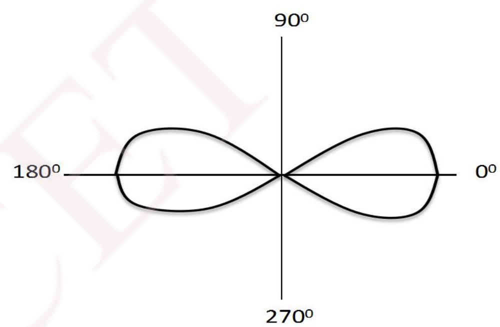


Fig: End fire array arrangement



Fig(a): Radiation pattern of Broad side array



Fig(b): Radiation pattern of End fire array

Two-element Broad side array (Equal amplitudes and same phase):

When all the elements of array supplied with currents of equal amplitude and same phase, then it fires the maximum radiation in perpendicular direction of the array axis and minimum radiation along the direction of the array axis. The structure of two element broad side array is shown in the figure below. The total field strength of two element broadside array is the vector sum of the fields of two individual elements.

The path difference between the weaves due to the two antennas is given by

$$P.d = dcos\theta \quad m$$

$$P.d = \frac{d}{\lambda}\cos\theta$$

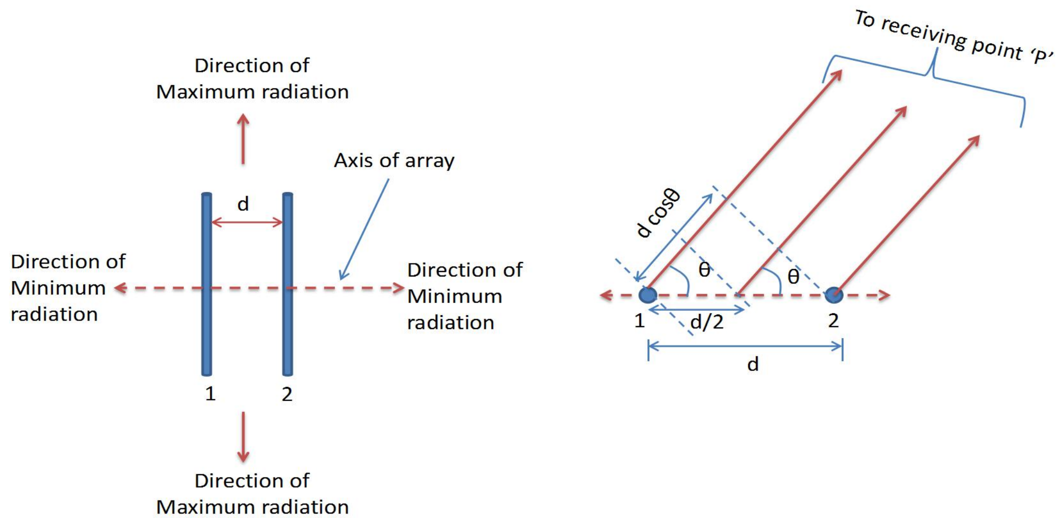


Fig: Two-element broad side array

Phase angle due to path difference is given by

$$\Psi = \beta d cos\theta \qquad -1$$

Total electric field at the interesting point (P) is given by

$$E = E_1 e^{-j\Psi/2} + E_2 e^{+j\Psi/2}$$

Where E_1 and E_2 are the magnitudes of electric filed strengths due to element 1 and 2 respectively.

Let $E_1 = E_2 = E_0$

$$E = E_0(e^{-j\Psi/2} + e^{+j\Psi/2})$$

$$E = E_0(e^{+j\Psi/2} + e^{-j\Psi/2})$$

Multiply R.H.S with 2/2

$$E = \frac{2}{2}E_0(e^{+j\Psi/2} + e^{-j\Psi/2})$$

$$E = 2E_0\left(\frac{e^{+j\Psi/2} + e^{-j\Psi/2}}{2}\right) = 2E_0 \cos(\Psi/2) \qquad -2$$

Substitute equation 1 in equation 2,

$$E = 2E_0 cos(\Psi/2) = 2E_0 cos(\beta d\, cos\theta/2) \qquad -3$$

The above equation represents the total electric field strength at the receiving point due to two element broad side array. In above equation the term '$2E_0$' represent the magnitude and cos(βd cosθ) represents the phase factor or pattern factor or array factor.

Direction of maximum radiation:

The direction of maximum radiation of major lobe can be obtained as follows:

We have

$$E = 2E_0 cos(\beta d\, cos\theta/2)$$

To have the maximum value for E, the pattern factor must be maximum. i.e.

$$\cos\left(\frac{\beta d\, cos\theta}{2}\right) = 1$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = 1$$

$$\cos\left(\frac{\pi d\, cos\theta}{\lambda}\right) = 1$$

To satisfy the above equation,

$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ n\pi$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{n\pi\lambda}{\pi d} = \pm\frac{n\lambda}{d}$$

$$\theta = \theta_{max} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) \qquad -4$$

Direction of minimum radiation:

The direction of minimum radiation of major lobe can be obtained as follows:

We have

$$E = 2E_0 cos(\beta d\, cos\theta/2)$$

To have the minimum value for E, the pattern factor must be minimum. i.e.

$$\cos\left(\frac{\beta d\, cos\theta}{2}\right) = 0$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = 0$$

$$\cos\left(\frac{\pi d\, cos\theta}{\lambda}\right) = 0$$

To satisfy the above equation,

$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ (2n+1)\pi/2$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{2\pi d} = \pm\frac{(2n+1)\lambda}{2d}$$

$$\theta = \theta_{min} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) \qquad -5$$

Direction of half power points:

The direction of half power points of major lobe can be obtained as follows:

We have

$$E = 2E_0 cos(\beta d\, cos\theta/2)$$

At half power points, E must be $1/\sqrt{2}$ times of maximum value

$$\cos\left(\frac{\beta d\, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi d\, cos\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$

To satisfy the above equation,

$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ (2n+1)\pi/4$$

Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{4\pi d} = \pm\frac{(2n+1)\lambda}{4d}$$

$$\theta = \theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) \qquad -6$$

The radiation pattern of 2-element broad side array with λ/2 spacing by using the above relations can be obtained as follows:

$$\theta_{max} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) = cos^{-1}\left(\pm\frac{n\lambda}{\frac{\lambda}{2}}\right) = cos^{-1}(\pm 2n)$$

When n = 0, $\theta_{max} = cos^{-1}(\pm 0) = 90^0 \ \& \ 270^0$

When n = 1, $\theta_{max} = cos^{-1}(\pm 2) = function\ not\ satisfied$

Similarly, $\theta_{min} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2\frac{\lambda}{2}}\right)$

$$\theta_{min} = cos^{-1}\left(\pm (2n+1)\right)$$

When n = 0, $\theta_{min} = cos^{-1}(\pm 1) = 0^0 \ \& \ 180^0$

When n = 1, $\theta_{min} = cos^{-1}(\pm 3) = not\ satisfied$

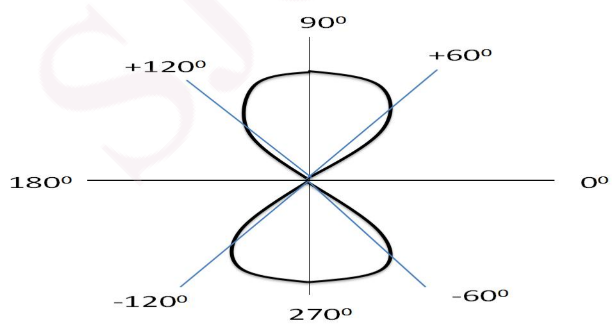
Similarly $\theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4\frac{\lambda}{2}}\right)$

$$\theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)}{2}\right)$$

When n = 0, $\theta_{HPP} = cos^{-1}\left(\pm\frac{1}{2}\right) = \pm 60^0 \ \& \pm 120^0 = +60^0, -60^0, +120^0 \ \& -120^0$

When n = 1, $\theta_{HPP} = cos^{-1}\left(\pm\frac{3}{2}\right) = not\ satisfied$

The resultant radiation pattern of 2 element broad side array with spacing λ/2 is shown in figure below.



Fig(a): Radiation pattern of Broad side array

Two-element end fire array (Equal amplitudes and opposite phase):

When all the elements of array supplied with currents of equal amplitude and opposite phase, then it fires the maximum radiation along the direction of the array axis and minimum radiation in perpendicular direction of the array axis. The structure of two element end-fire array is shown in the figure below. The total field strength of two element broadside array is the vector sum of the fields of two individual elements.

The path difference between the weaves due to the two antennas is given by

$$P.d = dcos\theta \quad m$$

$$P.d = \frac{d}{\lambda}\cos\theta$$

Phase angle due to path difference is given by

$$\Psi = \beta d cos\theta \qquad -1$$

Total electric field at the interesting point (P) is given by

$$E = -E_1 e^{-j\Psi/2} + E_2 e^{+j\Psi/2}$$

Where E_1 and E_2 are the magnitudes of electric filed strengths due to element 1 and 2 respectively. In above equation negative sign to E_1 represents the current of element 1 is opposite to the current of element 2.

Let $E_1 = E_2 = E_0$

$$E = E_0(-e^{-j\Psi/2} + e^{+j\Psi/2})$$

$$E = E_0(e^{+\frac{j\Psi}{2}} - e^{-j\Psi/2})$$

Multiply R.H.S with 2j/2j

$$E = \frac{2j}{2j}E_0(e^{+j\Psi/2} + e^{-j\Psi/2})$$

$$E = 2jE_0\left(\frac{e^{+j\Psi/2} + e^{-j\Psi/2}}{2j}\right) = 2jE_0 \sin(\Psi/2) \qquad -2$$

Substitute equation 1 in equation 2,

$$E = 2jE_0 sin(\Psi/2) = 2jE_0 sin(\beta d\ cos\theta/2) \qquad -3$$

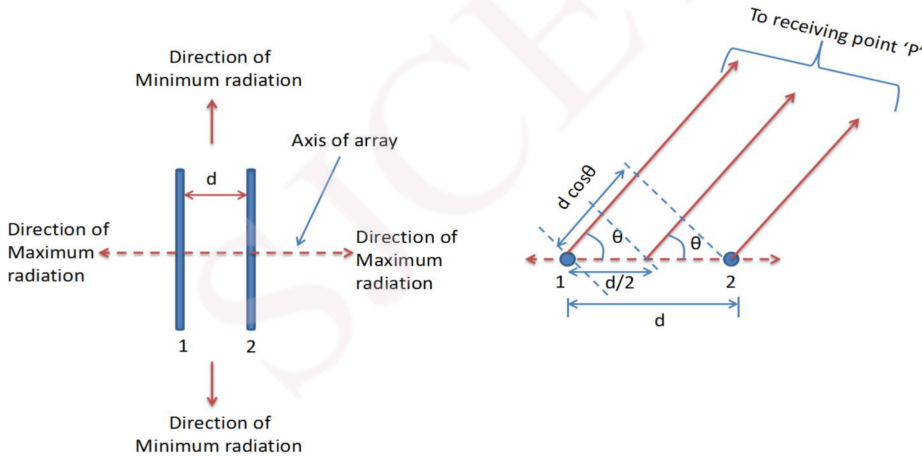


Fig: Two-element end fire array

The above equation represents the total electric field strength at the receiving point due to two element end fire array. In above equation the term '$2E_0$' represent the magnitude and sin(βd cosθ) represents the phase factor or pattern factor or array factor.

Direction of maximum radiation:

The direction of maximum radiation of major lobe can be obtained as follows:

We have

$$E = 2jE_0 sin(\beta d\ cos\theta/2)$$

To have the maximum value for E, the pattern factor must be maximum. i.e.

$$\sin\left(\frac{\beta d\ cos\theta}{2}\right) = 1$$

$$\sin\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = 1$$
$$\sin\left(\frac{\pi d\, cos\theta}{\lambda}\right) = 1$$
To satisfy the above equation,
$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ (2n+1)\pi/2$$
Where n = 0,1,2,3,…
$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{2\pi d} = \pm\frac{(2n+1)\lambda}{2d}$$
$$\theta = \theta_{max} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) \qquad -4$$
Direction of minimum radiation:
The direction of minimum radiation of major lobe can be obtained as follows:
We have
$$E = 2jE_0 sin(\beta d\, cos\theta/2)$$
To have the minimum value for E, the pattern factor must be minimum. i.e.
$$\sin\left(\frac{\beta d\, cos\theta}{2}\right) = 0$$
$$\sin\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = 0$$
$$\sin\left(\frac{\pi d\, cos\theta}{\lambda}\right) = 0$$
To satisfy the above equation,
$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ n\pi$$
Where n = 0,1,2,3,…
$$\cos\theta = \pm\frac{n\pi\lambda}{\pi d} = \pm\frac{n\lambda}{d}$$
$$\theta = \theta_{min} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) \qquad -5$$
Direction of half power points:
The direction of half power points of major lobe can be obtained as follows:
We have
$$E = 2jE_0 sin(\beta d\, cos\theta/2)$$
At half power points, E must be 1/√2 times of maximum value
$$\sin\left(\frac{\beta d\, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$
$$\sin\left(\frac{2\pi}{\lambda}\frac{d\, cos\theta}{2}\right) = \frac{1}{\sqrt{2}}$$
$$\sin\left(\frac{\pi d\, cos\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$
To satisfy the above equation,
$$\frac{\pi d\, cos\theta}{\lambda} = \pm\ (2n+1)\pi/4$$
Where n = 0,1,2,3,…

$$\cos\theta = \pm\frac{(2n+1)\pi\lambda}{4\pi d} = \pm\frac{(2n+1)\lambda}{4d}$$

$$\theta = \theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) \qquad -6$$

The radiation pattern of 2-element end fire array with λ/2 spacing by using the above relations can be obtained as follows:

$$\theta_{max} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2\frac{\lambda}{2}}\right) = cos^{-1}(\pm(2n+1))$$

When n = 0, $\theta_{max} = cos^{-1}(\pm 1) = 0^0$ & 180^0

When n = 1, $\theta_{max} = cos^{-1}(\pm 3) = not\ satisfied$

Similarly, $\theta_{min} = cos^{-1}\left(\pm\frac{n\lambda}{d}\right) = cos^{-1}\left(\pm\frac{n\lambda}{\frac{\lambda}{2}}\right) = cos^{-1}(\pm 2n)$

When n = 0, $\theta_{min} = cos^{-1}(\pm 0) = 90^0$ & 270^0

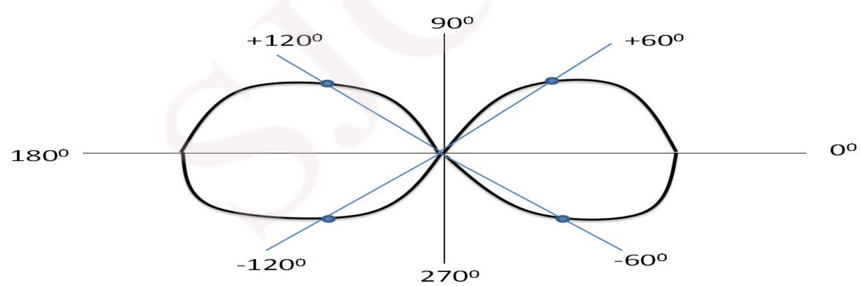
When n = 1, $\theta_{min} = cos^{-1}(\pm 2) = function\ not\ satisfied$

Similarly, $\theta_{HPP} = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4d}\right) = cos^{-1}\left(\pm\frac{(2n+1)\lambda}{4\frac{\lambda}{2}}\right) = cos^{-1}\left(\pm\frac{(2n+1)}{2}\right)$

When n = 0, $\theta_{HPP} = cos^{-1}\left(\pm\frac{1}{2}\right) = \pm 60^0$ & $\pm 120^0 = +60^0, -60^0, +120^0$ & -120^0

When n = 1, $\theta_{HPP} = cos^{-1}\left(\pm\frac{3}{2}\right) = not\ satisfied$

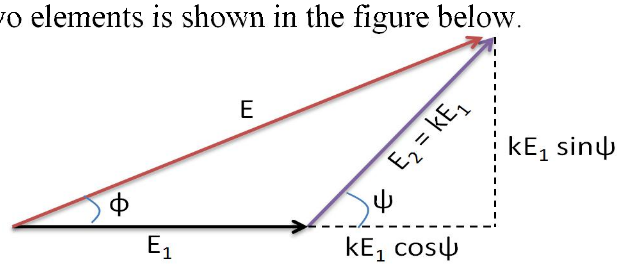
The resultant radiation pattern of 2 element end fire array with spacing λ/2 is shown in figure below.



Fig(b): Radiation pattern of End fire array

Two-element array with unequal amplitudes and any phase:

To find out the total electric field strength at any receiving point due to two-element array with unequal amplitudes and any phase, the vector addition is used. The vector diagram to add the field strengths due to two elements is shown in the figure below.



The total phase difference between waves due to element 1 and 2 is given by

Ψ = Phase angle due to path difference + phase angle difference between the input currents.

$$\psi = \beta d \cos\theta + \alpha$$

Total electric field strength at interesting point with reference to element 1 is given by

$$E = E_1 e^{j(0)} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi}$$

$$E = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right) = E_1\left(1 + k e^{j\psi}\right)$$

Where $k = \frac{E_2}{E_1}$

$$E = E_1\big(1 + k(\cos\psi + j\sin\psi)\big)$$

$$E = E_1(1 + k\cos\psi + jk\sin\psi) = E_1\big((1 + k\cos\psi) + jk\sin\psi\big)$$

$$|E| = E_1\sqrt{(1 + k\cos\psi)^2 + (jk\sin\psi)^2}$$

The phase angle ϕ is given by

$$\phi = \tan^{-1}\left(\frac{kE_1\sin\psi}{E_1 + kE_1\cos\psi}\right) = \tan^{-1}\left(\frac{k\sin\psi}{1 + k\cos\psi}\right)$$

UNIFORM LINEAR ARRAYS

Consider the uniform linear array contains N-no. of elements shown in the figure below.

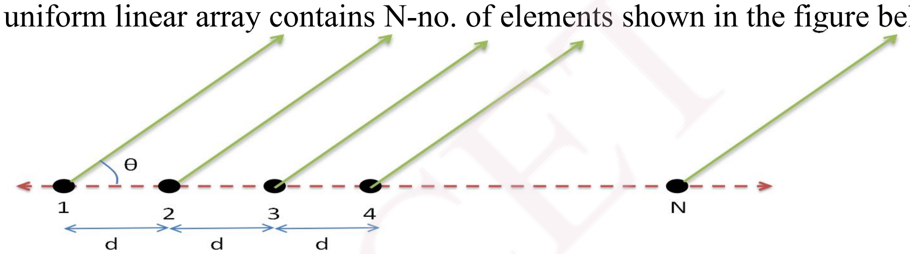


Fig: N-element linear array

Let us derive the general equation for the total electric field strength due to N-element linear array.

The total electric field strength (w.r.t element 1) at a point P is given by

$$E_t = E_1 e^{j0} + E_2 e^{j\Psi} + E_3 e^{j2\Psi} + E_4 e^{j3\Psi} + \ldots\ldots\ldots\ldots + E_N e^{j(N-1)\Psi} \qquad -1$$

Assume $E_1 = E_2 = E_3 = E_4 = \ldots\ldots\ldots E_N = E_0$

Then equation 1 becomes

$$E_t = E_0\left(1 + e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots\ldots + e^{j(N-1)\Psi}\right) \qquad -2$$

Multiply both side with $e^{j\Psi}$

$$E_t e^{j\Psi} = e^{j\Psi} E_0\left(1 + e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots\ldots + e^{j(N-1)\Psi}\right)$$

$$E_t e^{j\Psi} = E_0(e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + e^{j4\Psi} \ldots\ldots + e^{jN\Psi}) \qquad -3$$

Subtract equation 3 from 2,

$$E_t(1 - e^{j\Psi}) = E_0(1 - e^{jN\Psi})$$

$$E_t = \frac{E_0(1 - e^{jN\Psi})}{(1 - e^{j\Psi})}$$

$$E_t = \frac{E_0\left(e^{jN\psi/2}.e^{-jN\psi/2} - e^{jN\psi/2}.e^{jN\psi/2}\right)}{e^{j\psi/2}.e^{-j\psi/2} - e^{j\psi/2}.e^{j\psi/2}} = \frac{-E_0 e^{jN\psi/2}\left(-e^{-jN\psi/2} + e^{jN\psi/2}\right)}{-e^{j\psi/2}\left(-e^{-j\psi/2} + e^{j\psi/2}\right)}$$

$$E_t = \frac{E_0 e^{jN\psi/2}\left(e^{jN\psi/2} - e^{-jN\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)} \qquad -4$$

We know that,

$$\sin(N\psi/2) = \frac{e^{jN\psi/2} - e^{-jN\psi/2}}{2j}$$

$$e^{jN\psi/2} - e^{-jN\psi/2} = 2j\sin(N\psi/2) \qquad -5$$

Similarly $\sin(\psi/2) = \frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}$

$$e^{j\psi/2} - e^{-j\psi/2} = 2j\sin(\psi/2) \qquad -6$$

Substitute equations 5 and 6 in equation 4

$$E_t = \frac{E_0 e^{jN\psi/2}(2j\sin(N\psi/2))}{e^{j\psi/2}(2j\sin(\psi/2))} = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

Wher $\emptyset = (N-1)\Psi/2$

In above equation the factor $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ is called array factor or pattern factor.

N-Element Broadside Arrays:

When all the elements of array supplied with currents of equal amplitude and same phase, then it fires the maximum radiation in perpendicular direction of the array axis and minimum radiation along the direction of the array axis.

Direction of maximum radiation of minor lobes or pattern maxima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

To have the maximum value for E, the pattern factor must be maximum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be maximum. To have maximum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be maximum.

$$\sin\left(\frac{N\Psi}{2}\right) = 1$$

To satisfy the above relation

$$\frac{N\psi}{2} = \pm\ (2n+1)\pi/2$$

Where n = 1,2,3,…

n = 0 for major lobe

$$N\psi = \pm\ (2n+1)\pi$$

$$\psi = \pm \frac{(2n+1)\pi}{N}$$

But $\psi = \beta d\cos\theta + \alpha$

$$\beta d\cos\theta + \alpha = \pm \frac{(2n+1)\pi}{N}$$

But α = 0 for broad side array

$$\beta d\cos\theta = \pm \frac{(2n+1)\pi}{N}$$

$$\cos\theta = \pm \frac{(2n+1)\pi}{\beta dN} = \pm \frac{(2n+1)\pi}{\frac{2\pi}{\lambda} dN}$$

$$\cos\theta = \pm \frac{(2n+1)\lambda}{2Nd}$$

$$(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}\right) \qquad -7$$

Direction of minimum radiation of minor lobes or pattern minima:

The total electric field strength due to N-element broad side array is given by

$$E_t = E_0 \frac{\sin(N\Psi/2)}{\sin(\Psi/2)} e^{j\emptyset}$$

To have the minimum value for E, the pattern factor must be minimum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be minimum. To have minimum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be minimum.

$$\sin\left(\frac{N\Psi}{2}\right) = 0$$

To satisfy the above relation

$$\frac{N\psi}{2} = \pm\, n\pi$$

Where n = 1,2,3,… n = 0 for major lobe

$$N\psi = \pm\, 2n\pi$$

$$\psi = \pm\frac{2n\pi}{N}$$

But

$$\psi = \beta d\cos\theta + \alpha$$

$$\beta d\cos\theta + \alpha = \pm\frac{(2n+1)\pi}{N}$$

But α = 0 for broad side array

$$\beta d\cos\theta = \pm\frac{2n\pi}{N}$$

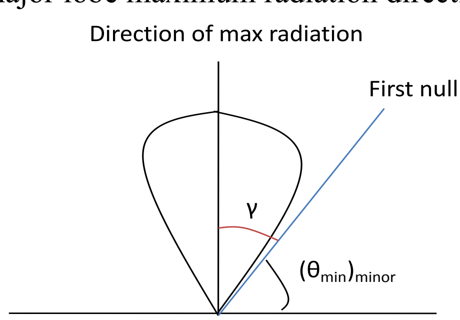
$$\cos\theta = \pm\frac{2n\pi}{\beta dN} = \pm\frac{2n\pi}{\frac{2\pi}{\lambda}dN}$$

$$\cos\theta = \pm\frac{n\lambda}{Nd}$$

$$(\theta_{min})_{minor} = \cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right)$$

Beam width of major lobe:

Beam width or BWFN of a major lobe is defined as the angle between the first nulls or twice the angle between first null and major lobe maximum radiation direction as shown in figure below.



Let the angle between first null and major lobe maximum radiation direction is γ

From figure, $(\theta_{min})_{minor} = 90 - \gamma$

$$90-\gamma=\cos^{-1}\left(\frac{n\lambda}{Nd}\right)$$
$$\cos(90-\gamma)=\frac{n\lambda}{Nd}$$
$$\sin\gamma\cong\gamma=\frac{n\lambda}{Nd}$$
Replace n with 1, because first null occurs when n = 1
$$\gamma=\frac{\lambda}{Nd}$$
$$Beam\ width=BWFN=2\gamma=\frac{2\lambda}{Nd}\quad in\ radians$$
$$Beam\ width=BWFN=\frac{2\lambda}{Nd}\times 57.3^0=\frac{114.6\ \lambda}{Nd}\quad in\ degrees$$
Half Power Beam Width of N-element Broad side array is given by
$$\text{HPBW}=\frac{BWFN}{\mathbf{2}}=\frac{57.3\ \lambda}{\text{Nd}}\ \text{degree}$$

End-fire Arrays:

When all the elements of array supplied with currents of equal amplitude and opposite phase, then it fires the maximum radiation in the direction of the array axis and minimum radiation in perpendicular direction of the array axis.

Let us get small equation for α.

To have a maximum value of electric field strength at any direction, the total phase angle ψ must be zero. In case of end fire array, the maximum radiation will be at 0^0 and 180^0 directions. Therefore
$$\psi=\beta d\cos\theta+\alpha=0$$
Let maximum radiation direction is 0^0. i.e $\theta=0^0$
$$\beta d\cos(0)+\alpha=0$$
$$\beta d+\alpha=0$$
$$\alpha=-\beta d\qquad -1$$

Direction of maximum radiation of minor lobes or pattern maxima:

The total electric field strength due to N-element broad side array is given by
$$E_t=E_0\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}e^{j\emptyset}$$
To have the maximum value for E, the pattern factor must be maximum. That is
$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$
must be maximum. To have maximum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be maximum.
$$\sin\left(\frac{N\Psi}{2}\right)=1$$
To satisfy the above relation $\frac{N\psi}{2}=\pm\ (2n+1)\pi/2$

Where n = 1,2,3,… n = 0 for major lobe
$$N\psi=\pm\ (2n+1)\pi$$
$$\psi=\pm\frac{(2n+1)\pi}{N}$$
But $\psi=\beta d\cos\theta+\alpha$

$$\beta d\cos\theta+\alpha=\pm\frac{(2n+1)\pi}{N} \qquad -2$$

Substitute equation 1 in equation2

$$\beta d\cos\theta-\beta d=\pm\frac{(2n+1)\pi}{N}$$

$$\beta d(\cos\theta-1)=\pm\frac{(2n+1)\pi}{N}=$$

$$\cos\theta-1=\pm\frac{(2n+1)\pi}{\beta dN}=\pm\frac{(2n+1)\pi}{\frac{2\pi}{\lambda}dN}$$

$$\cos\theta-1=\pm\frac{(2n+1)\lambda}{2Nd}$$

$$\cos\theta=\pm\frac{(2n+1)\lambda}{2Nd}+1$$

$$(\theta_{max})_{minor}=\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}+1\right) \qquad -3$$

Direction of minimum radiation of minor lobes or pattern minima:

The total electric field strength due to N-element broad side array is given by

$$E_t=E_0\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}e^{j\emptyset}$$

To have the minimum value for E, the pattern factor must be minimum. That is

$$\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$$

must be minimum. To have minimum value for $\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}$ its numerator must be minimum.

$$\sin\left(\frac{N\Psi}{2}\right)=0$$

To satisfy the above relation $\frac{N\psi}{2}=\pm\ n\pi$

Where n = 1,2,3,… n = 0 for major lobe

$$N\psi=\pm\ 2n\pi$$

$$\psi=\pm\frac{2n\pi}{N}$$

But $\psi=\beta d\cos\theta+\alpha$

$$\beta d\cos\theta+\alpha=\pm\frac{(2n+1)\pi}{N} \qquad -4$$

Substitute equation 1 in equation4

$$\beta d\cos\theta-\beta d=\pm\frac{2n\pi}{N}$$

$$\beta d(\cos\theta-1)=\pm\frac{2n\pi}{N}$$

$$\cos\theta-1=\pm\frac{2n\pi}{\beta dN}=\pm\frac{2n\pi}{\frac{2\pi}{\lambda}dN}$$

$$\cos\theta-1=\pm\frac{n\lambda}{Nd}$$

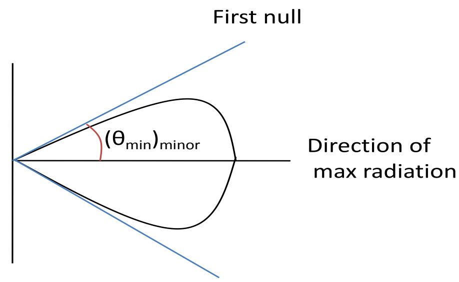
But $\cos\theta=1-2sin^2\theta/2$

$$1 - 2sin^2\theta/2 - 1 = \pm\frac{n\lambda}{Nd}$$

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right) \qquad -5$$

Beam width of major lobe:

The beam width can be obtained from the following figure



$$\text{Beam width (BWFN)} = 2 \text{ X } (\theta_{min})_{minor} \qquad -6$$

$$(\theta_{min})_{minor} = 2\sin^{-1}\left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$\frac{(\theta_{min})_{minor}}{2} = \sin^{-1}\left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$\sin\left(\frac{(\theta_{min})_{minor}}{2}\right) = \left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

When $(\theta_{min})_{minor}$ is small, then

$$\sin\left(\frac{(\theta_{min})_{minor}}{2}\right) \cong \left(\frac{(\theta_{min})_{minor}}{2}\right) = \left(\sqrt{\frac{n\lambda}{2Nd}}\right)$$

$$(\theta_{min})_{minor} = 2\sqrt{\frac{n\lambda}{2Nd}} = \sqrt{\frac{2n\lambda}{Nd}} \qquad -7$$

Substitute equation 7 in equation 6

$$BWFN = 2\sqrt{\frac{2n\lambda}{Nd}}$$

Replace n with 1, because first null occurs when n = 1

Beam Width between First Nulls (BWFN) of N-element end-fire array is given by

$$\text{BWFN} = 2\sqrt{\frac{2\lambda}{\text{Nd}}}\text{ radian} = 2\sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3 \text{ degree}$$

Half Power Beam Width of N-element end-fire array is given by

$$\text{HPBW} = \sqrt{\frac{2\lambda}{\text{Nd}}}\,\text{radian} = \sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3 \ \text{degree}$$

COMPARISON OF DIFFERENT ARRAY:

Comparison of various arrays is given in the table below:

S.No	Parameter	Broad side array	Endfire array	EFA with increased directivity
1	HPBW	$\frac{57.3\,\lambda}{\text{Nd}}$ degree	$\sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3$ degree	$\sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3$ degree
2	BWFN	$\frac{114.6\,\lambda}{\text{Nd}}$ degree	$2\sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3$ degree	$2\sqrt{\frac{2\lambda}{\text{Nd}}} \times 57.3$ degree
3	Direction of minor lobes maxima	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd}\right)$	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd} + 1\right)$	$\cos^{-1}\left(\pm\frac{(2n+1)\lambda}{2Nd} + 1\right)$
4	Direction of minor lobes minima	$\cos^{-1}\left(\pm\frac{n\lambda}{Nd}\right)$	$2\cos^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$	$2\cos^{-1}\left(\pm\sqrt{\frac{n\lambda}{2Nd}}\right)$
5	Directivity	$D = \frac{2\text{Nd}}{\lambda}$	$D = \frac{4\text{Nd}}{\lambda}$	$D = 1.789\left(\frac{4\text{Nd}}{\lambda}\right)$

BINOMIAL ARRAYS:

The major disadvantage of linear arrays such as broad side or endfire array is, the number of minor lobes will be increases when the distance between the elements (d) or number of elements (N) increases. This drawback can be avoided by using the binomial array .i.e. number of minor lobes can be reduced by using the binomial array. In binomial array, non uniform amplitudes will be applied to the individual elements. In this array, the amplitudes of the radiating sources are arranged according to the coefficients of successive terms of the following binomial series and hence the name.

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!}a^{n-2}b + \frac{(n-1)(n-2)}{2!}a^{n-3}b^2 + \frac{(n-1)(n-2)(n-3)}{3!}a^{n-4}b^3 + \ldots.$$

Where n is the number of elements in the array.

The following two conditions were satisfied in binomial array to reduce the number of minor lobes.

(i) Spacing between the two consecutive radiating sources does not exceed λ/2.

(ii) The current amplitudes in radiating sources are proportional to the coefficients of the successive terms of the binomial series.

The coefficients of successive terms can be obtained either from the binomial series or from the Pascal's triangle shown in the figure below.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

Fig: Pascal's Triangle

Advantages of Binomial array:

(i) Less number of minor lobes as compared with linear arrays

(ii) Beam width increases

Disadvantages of Binomial array:

(i) Beam width increases and hence the directivity will be decreases

(ii) For design of large array, larger amplitude ratio of sources is required.

PRINCIPLE OF PATTERN MULTIPLICATION

The principle of pattern multiplication or multiplication of patterns is stated as follows:

"The total field pattern of array of non-isotropic but similar sources is the multiplication of the individual source pattern and the pattern of array of isotropic point sources each located at the phase center of individual source, and having the relative amplitude and phase, where as the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources". This statement can be expressed as

$$E = \{E_i(\theta,\varphi) \times E_a(\theta,\varphi)\} \times \{E_{pi}(\theta,\varphi) \times E_{pa}(\theta,\varphi)\}$$

Where $E_i(\theta,\varphi)$ is the field or magnitude pattern of individual source

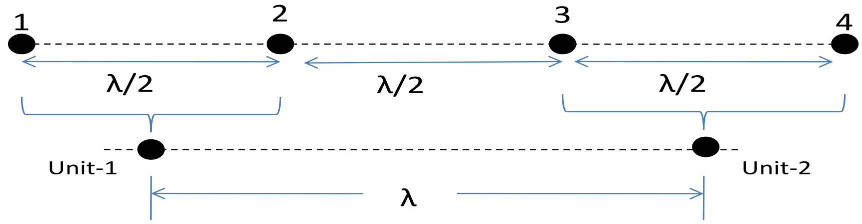
$E_a(\theta,\varphi)$ is field or magnitude pattern of array of isotropic point sources

$E_{pi}(\theta,\varphi)$ is the phase pattern of individual source

$E_{pa}(\theta,\varphi)$ is the phase pattern of array of isotropic point sources.

The pattern multiplication has great advantage that, it makes possible to sketch rapidly, almost by inspection, the pattern of complicated arrays.

Let us explain the concept of pattern multiplication by considering the 4 element broadside array with spacing λ/2 as shown in figure below.



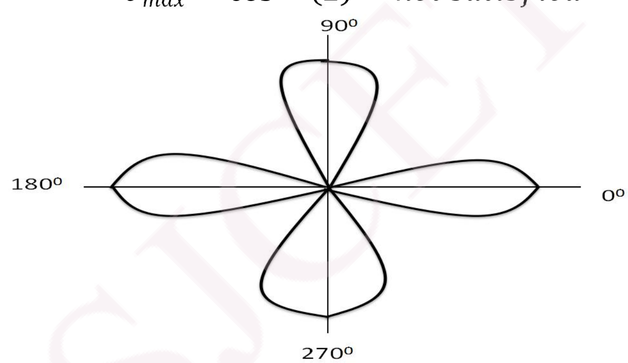
The pattern of elements 1 and 2 operating as a unit, that is two antennas spaced λ/2 and fed in phase (broad side array). Also antennas 2 and 3 will be considered as another unit with the same pattern of figure of eight shape. Now the 4-element array has been reduced to 2-element (units) array with spacing λ as shown figure. The radiation pattern of two element broad side array with spacing λ can be obtained by using the two element array analysis and is shown in the figure below. This pattern is called group pattern.
From the two element array analysis, we know that

$$\theta_{max} = \cos^{-1}\left(\pm\frac{n\lambda}{d}\right) = \cos^{-1}\left(\pm\frac{n\lambda}{\lambda}\right) = \cos^{-1}(\pm n)$$

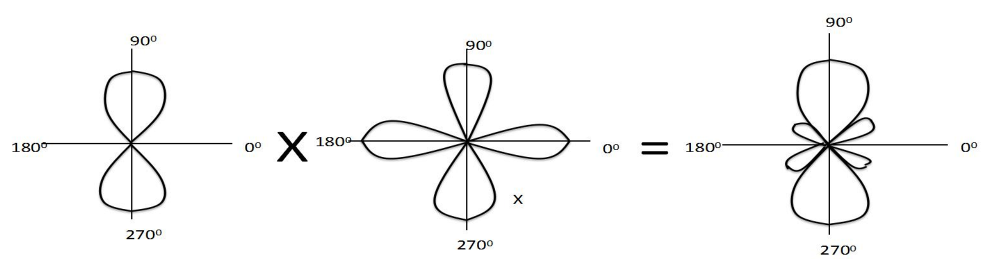
When n = 0, $\theta_{max} = \cos^{-1}(0) = 90^0 \;\&\; 270^0$

When n = 1, $\theta_{max} = \cos^{-1}(1) = 0^0 \;\&\; 180^0$

When n = 2, $\theta_{max} = \cos^{-1}(2) = not\ satisfied$

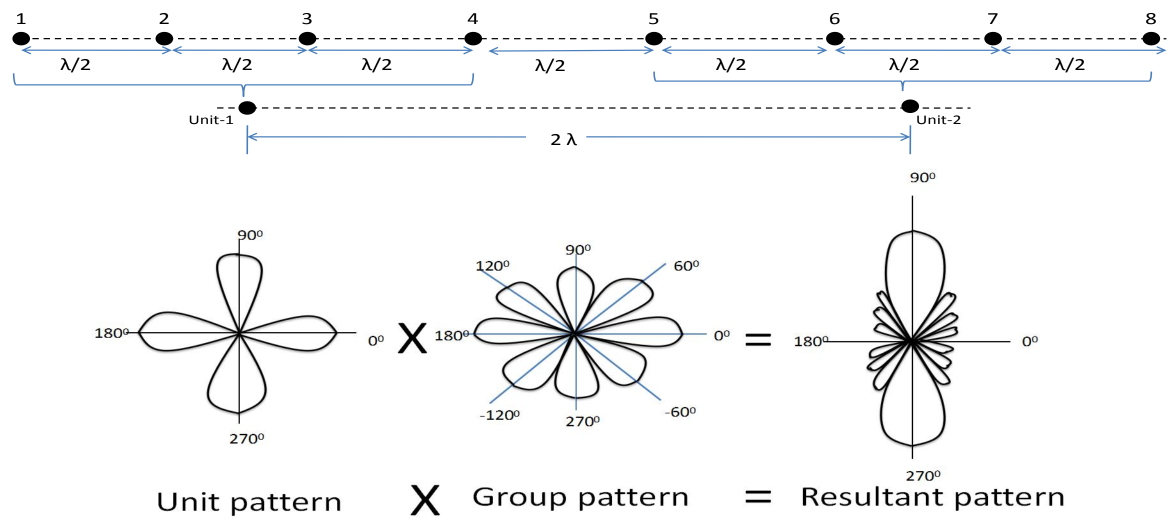


The resultant pattern for the original 4-element array is obtained by multiplying the unit pattern with group pattern. The above procedure can be represented in the following figure below.



Unit pattern X Group pattern = Resultant pattern

Similarly the radiation pattern of 8 element broad side array with spacing λ/2 is shown in the figure below.



PARASITIC ARRAY(YAGI-UDA ARRAY)

Yagi - Uda Arrays:

Array is defined as the method of combining the radiations from the group or array of antennas by involving the wave interference. Parasitic array or 'array with parasitic elements' is an array which contains one driven element and number of parasitic or passive elements. Example of parasitic array is the Yagi-Uda array. The Yagi-Uda array is invented by S.Uda and H.Yagi. The structure of 3-element yagi uda array is shown in the figure below.

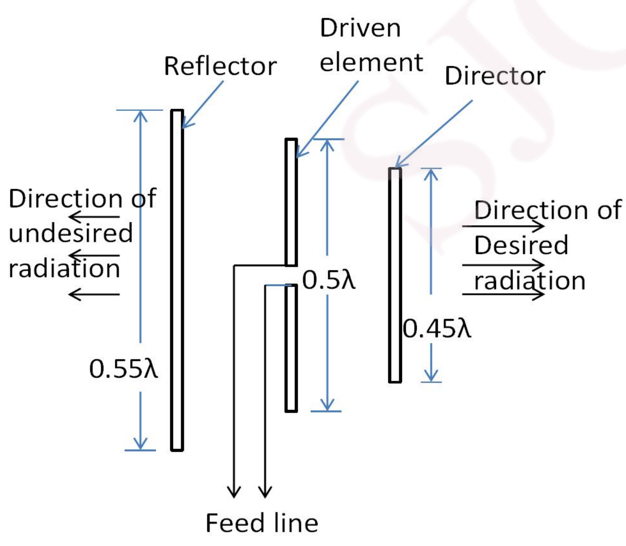


Fig : 3-element yagi-uda array

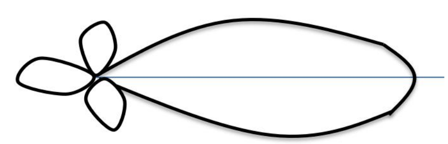


Fig : Radiation pattern

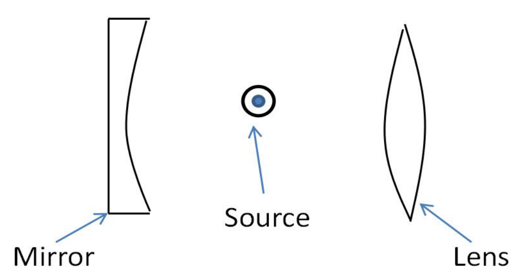


Fig : Optical equivalent

The radiation pattern and optical equivalent also shown in the figure. The principle of operation of the yagi-uda array can be explained as follows:

(i) 3-element Yagi-Uda antenna consists of one driven element, one reflector and one director.

(ii) The input signal will be supplied to the driven element and the two passive elements (reflector and director) are parasitically or electromagnetically coupled to the driven element.

(iii) The function of the reflector is to reflect back the signal and the function of the director is to further forward the signal in the forward direction.

(iv) The reflector having the nature of inductive where as the director having the nature of capacitive. The reason for this is the length of the reflector is greater than the driven element and the length of the director is smaller than the driven element.

(v) By selecting the proper length of the elements and proper spacing between the elements we can produce the highly directional beam.

(vi) The Yagi-Uda antenna is a hi9gh gain antenna. It provides the gain in the order of 8 dB and Front to Back Ratio (FBR) of about 20 dB.

(vii) To achieve the greater directivity, the number directors can be increased.

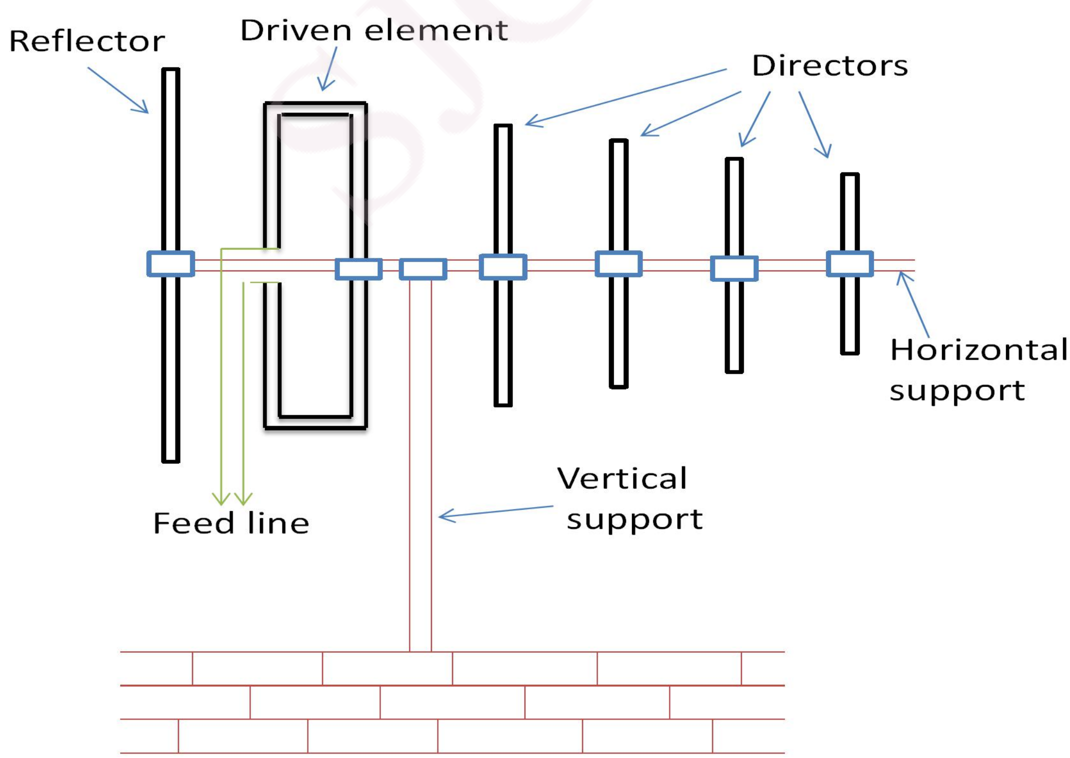
(viii) The approximate formulae for the length of the driven element, reflector and director is given by

Length of the reflector = 500/f(MHz) feet

Length of the driven element = 475/f(MHz) feet

Length of the director = 455/f(MHz) feet

The practical structure of 6-element Yagi-Uda antenna is shown in the figure below.



Folded Dipoles &their characteristics:

A very important variation of conventional half wave dipole is the folded dipole which is shown in the figure below.

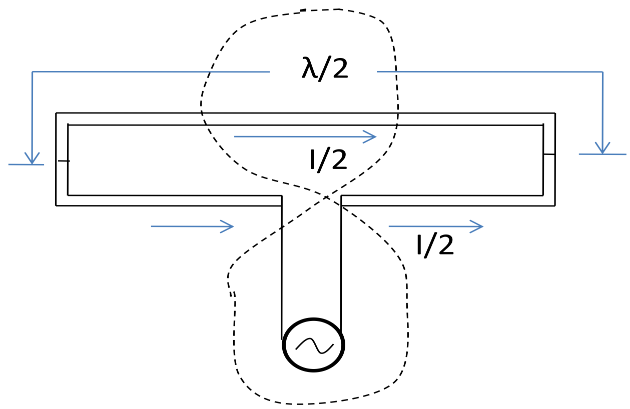


Fig : Folded dipole with current distribution and radiation pattern

The following are the important points to be noted about the folded dipole:

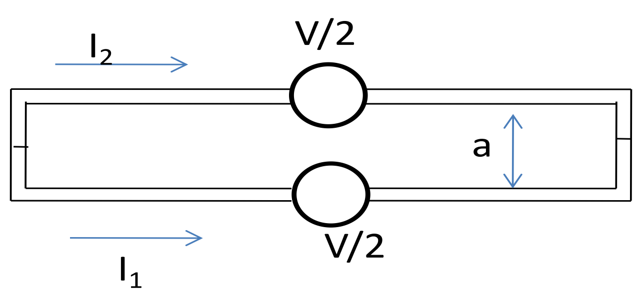
(i) Folded dipole is a combination of two half wave dipoles, one is continuous and other is splitted at the center.

(ii) The advantages of folded dipole as compared with conventional half wave dipole are high input impedance, wide band in frequency and act as a built in reactance compensations network.

(iii) The shape of radiation pattern due to the folded dipole is figure of eight shape or doughnut shape.

(iv) The folded dipole with conductors of equal radius can provide impedance up to 292 ohms and folded tripole (combination of three half wave dipoles) can provide impedance up to 657 ohms.

The equation for the input impedance of the folded dipole is obtained from the following figure:



$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12}$$

But $I_1 = I_2$ because the two conductors are in series.

$$\frac{V}{2} = I_1 Z_{11} + I_1 Z_{12} = I_1(Z_{11} + Z_{12})$$

When the spacing between the two conductors is small, then $Z_{11} = Z_{12}$

$$\frac{V}{2} = I_1(Z_{11} + Z_{11}) = 2I_1Z_{11}$$

$$Z = \frac{V}{I_1} = 4Z_{11} = 4(73) = 292\ Ohms.$$

In general

$$Z = n^2 Z_{11}$$

Where n represents the number of half wave dipoles used.

When the folded dipole is made with conductors of unequal radii, then the input impedance can be obtained by the following formula.

$$Z = Z_{11}\left(1 + \frac{r_2}{r_1}\right)^2$$

Where, r_2 and r_1 represent the radii of conductors.

HELICAL ANTENNAS

Helical Geometry:

The geometry of constructional features of helical antenna is shown in the figure below.

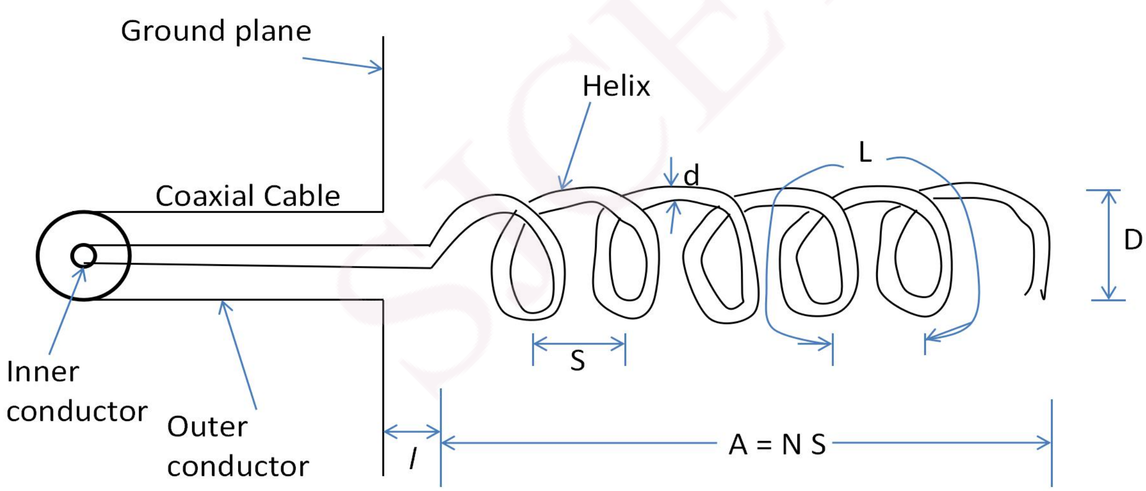


Fig : Helical antenna

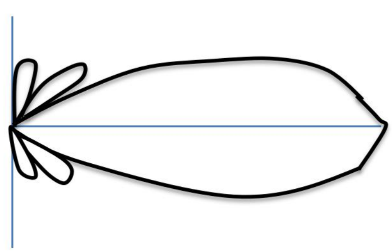
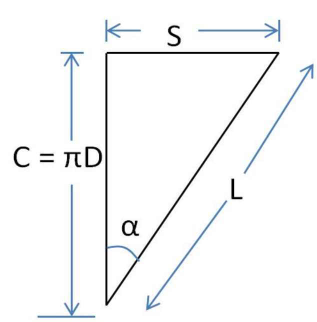


Fig : Radiation pattern



Helical antenna consists of a helix of thick copper wire or tubing wound in the shape of a screw thread and used as an antenna in conjunction with a flat metal plate called a ground plate. It is the simplest antenna to produce the circularly polarized waves. It is a broadband antenna. The physical parameters of the helical antenna are listed below:

C = Circumference of the helix
d = Diameter of the helix conductor
D = Diameter of the helix
A = NS = Axial length or length of the helix
N = Number of turns
S = Spacing between the turns
L = Length of each turn
l = Spacing between the helix and the ground plate.
$\alpha = \tan^{-1}(S/C)$ = Pitch angel of the helix.

The pitch angle (α) of the helix is defined as the angle between the line Tangent to the helix wire and the plane normal to the helix axis.

<u>Helix modes:</u>

Helical antenna can be operated in two modes such as

(i) Normal mode or perpendicular mode of operation
(ii) Axial mode or beam mode of radiation

Normal Mode: The helical antenna can be operated in the normal mode when the dimensions of the helix are smaller than the wavelength (λ). The helical antenna is a combination of loop antenna and short dipole as shown in the figure below.

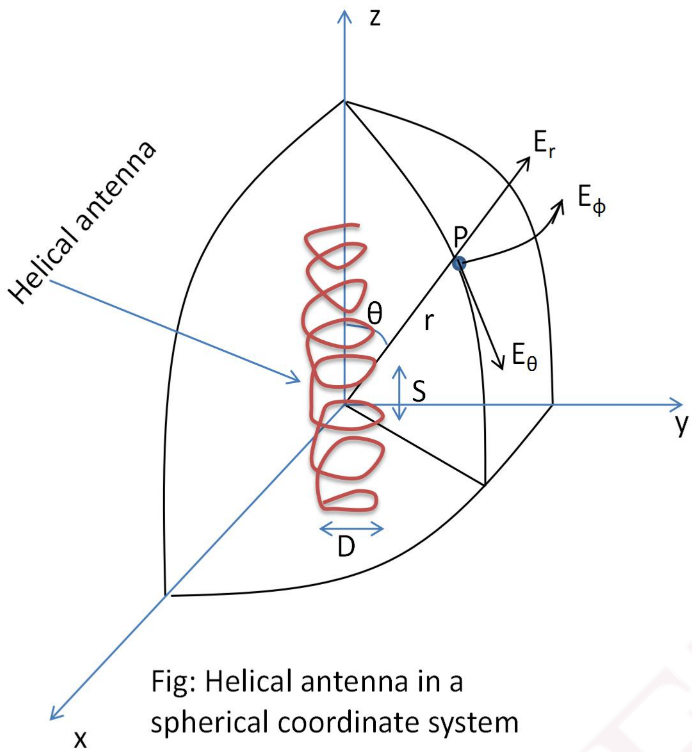


Fig: Helical antenna in a spherical coordinate system

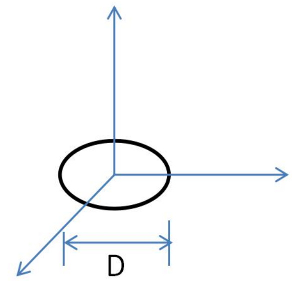


Fig : Loop when α = 0^0

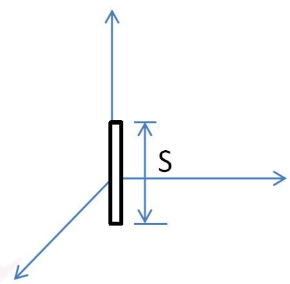


Fig : Short dipole when α = 90^0

In this mode of operation, the direction of maximum radiation is in perpendicular or in normal direction with respect to the helix axis. The shape of the radiation pattern in this mode is bidirectional. When the spacing 'S' of the helical antenna tends to zero or when the pitch angle is equal to 0^0, then the helix reduces to loop antenna as shown in the figure above. Similarly when the diameter 'D' of the helix tends to zero or pitch angle is equal to 90^0, then the helical antenna reduced to short dipole with length 'S'. Therefore the far field equations of the helical antenna van be obtained from the far field equations of short dipole and loop antenna.
The electric filed component due to the short dipole of length 'S' is given by

$$E_\theta = \frac{j60\pi[I]sin\theta}{r}.\frac{S}{\lambda}$$

Similarly the electric field component due to the loop antenna is given by

$$E_\emptyset = \frac{120\pi^2[I]sine\theta}{r}.\frac{A}{\lambda^2}$$

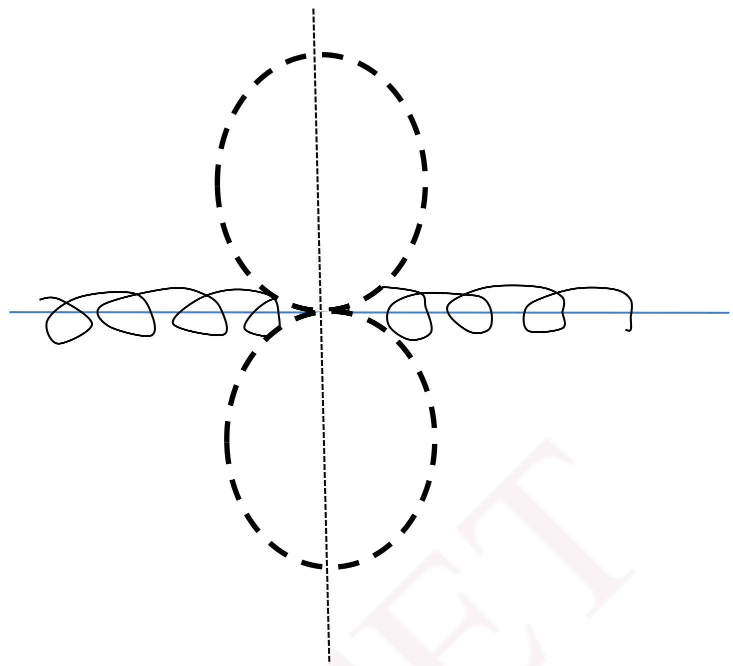
Where A = area of the loop antenna.
For circular loop with diameter 'D' the area is given by $A = \pi D^2/4$
The axial ratio (AR) of helical antenna is given by

$$AR = \frac{|E_\theta|}{E_\emptyset} = \frac{S\lambda}{2\pi A} = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

AR = 1, for circular polarization
AR > 1, for elliptical polarization
AR = 0, for linear horizontal polarization

AR = ∞, for linear vertical polarization

The radiation pattern of the helical antenna in the normal mode is shown in the figure below.



Axial Mode: The helical antenna can be operated in the axial mode when the dimensions of the helix are larger than the wavelength (λ). Axial mode is also known as beam mode of radiation. In this mode the maximum radiation will be in the direction of the helix axis. The radiation pattern produced in this mode is unidirectional. This mode of operation is preferable used to produce the circular polarization. The radiation pattern of the helical antenna under the axial mode is shown in the figure below.

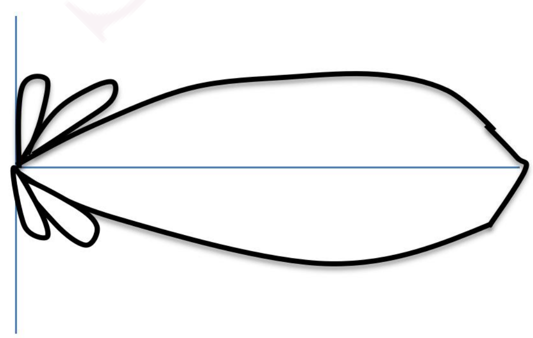


Fig : Radiation pattern

<u>Practical Design considerations for Monofilar Helical Antenna in Axial and Normal Modes:</u>
When the helical antenna is made from the single conductor, then it is known as monofilar helical antenna. Practical design considerations of monofilar helical antenna are given below.

Axial length (A) = N S
Spacing between the helix and ground plane (l) = S/2
Length of each turn (L) = $\sqrt{S^2 + C^2}$
Circumference of the helix (C) = π D
Pitch angel (α) = $\tan^{-1}$(S/C)
Axial Ratio (AR) = $2S\lambda/C^2$
Half Power Beam Width (HPBW) = $\frac{52}{C}\sqrt{\frac{\lambda^3}{NS}}$
Beam Width between First Nulls (BWFN) = $\frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}$
Directivity (D) = $\frac{15\,NSC^2}{\lambda^3}$

SOLVED PROBLEMS

1. A dipole antenna with length equal to 25 cm and carrying a current of 2 A at a frequency of 8.5 MHz radiates in to free space. Calculate the total power radiated by that antenna.
Sol:
Given data:

$$lenght\ of\ the\ dipole\ (l) = 25\ cm = 0.25\ m$$
$$Current\ (I_m) = 2\ A$$
$$Frequency\ (f) = 8.5\ MHz, \lambda = \frac{c}{f} = \frac{3 \times 10^8}{8.5 \times 10^6} = 35.294\ m$$

The power radiated due to short dipole is given by

$$W = \frac{\eta\left((\beta I_m l)\right)^2}{12\pi} = \frac{\eta\beta^2 I_m^2 l^2}{12\pi} = \frac{\eta\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{12\pi}$$
$$W = \frac{120\pi \times 4\pi^2 \times 2^2 \times 0.25^2}{12\pi \times 35.294} = 0.0792\ watts$$

2. At what frequency the 65 cm length antenna produces a radiation resistance of 0.75 Ω
Sol:
Given data:

$$length\ (l) = 65\ cm = 0.65\ m$$
$$Radiation\ resistance (R_r) = 075\ \Omega$$

We know that,

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\frac{l^2}{\lambda^2}$$
$$\lambda^2 = \frac{80\,\pi^2 l^2}{R_r}$$
$$\lambda = \sqrt{\frac{80\,\pi^2 l^2}{R_r}}$$

But λ = c/f

$$\frac{c}{f} = \sqrt{\frac{80\,\pi^2 l^2}{R_r}}$$

$$f = c\sqrt{\frac{R_r}{80\,\pi^2 l^2}} = 3 \times 10^8 \sqrt{\frac{0.75}{80\,\pi^2 \times 0.65^2}} = 14.3\ MHz$$

3. Calculate the radiation resistance of a dipole antenna having length λ/8, if the equivalent loss resistance accounting for the heat loss in the antenna due to finite conductivity of the dipole is 1.5 Ω. Also find the efficiency of the antenna.

Sol:

Given data:

$$Length\ of\ the\ dipole(l) = \frac{\lambda}{8}$$

$$Loss\ resistance\ (R_l) = 1.5\ \Omega$$

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\frac{l^2}{\lambda^2}$$

$$R_r = 80\pi^2\frac{\left(\frac{\lambda}{8}\right)^2}{\lambda^2} = 98.75\ \Omega$$

Antenna efficiency is given by

$$\%\,\eta = \frac{R_r}{R_r + R_l} \times 100 = \frac{98.75}{98.75 + 1.5} \times 100 = 98.5\%$$

4. A radiating element of 1 cm carries an effective current of 0.5 Amp at 3 GHz. Calculate the radiated power

Sol:

Length of the element (l) = 1 cm = 0.01 m

Current (I_m) = 0.5 Amp

Frequency (f) = 3 GHz

Wavelength is given by

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1m$$

The radiated power is given by

$$W = \frac{\eta(\beta I_m l)^2}{12\pi} = \frac{\eta\beta^2 I_m^2 l^2}{12\pi} = \frac{\eta\left(\frac{2\pi}{\lambda}\right)^2 I_m^2 l^2}{12\pi}$$

$$W = \frac{120\pi\times\left(\frac{2\pi}{0.1}\right)^2\times(0.5)^2\times(0.01)^2}{12\pi} = 0.986w$$

5. Design 3 element Yagi-Uda array with frequency of operation 64 MHz.

Sol:

$$Length\ of\ the\ reflector = \frac{500}{f(MHz)}\quad feet = \frac{500}{64} = 7.812\ feet$$

$$Length\ of\ the\ driven\ element = \frac{475}{f(MHz)}\ feet = \frac{475}{64} = 7.421\ feet$$

$$Length\ of\ the\ director = \frac{455}{f(MHz)}\ feet = \frac{455}{64} = 7.11\ feet$$

6. Design Yagi-Uda antenna of six elements to provide a gain of 12 dBi if the operating frequency is 200 MHz.

Sol:

Given

Frequency (f) = 200 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5\ m$$

$$Length\ of\ reflector = 0.475\ \lambda = 0.475 \times 1.5 = 0.7125\ m$$

$$Length\ of\ driven\ element = 0.46\ \lambda = 0.46 \times 1.5 = 0.69\ m$$

$$Length\ of\ first\ director = 0.44\ \lambda = 0.44 \times 1.5 = 0.66\ m$$

$$Length\ of\ second\ director = 0.44\ \lambda = 0.44 \times 1.5 = 0.66\ m$$

$$Length\ of\ third\ director = 0.43\ \lambda = 0.43 \times 1.5 = 0.645\ m$$

$$Length\ of\ fourth\ director = 0.40\ \lambda = 0.40 \times 1.5 = 0.6\ m$$

$$Spacing\ between\ reflector\ and\ driven\ element = 0.25\ \lambda = 0.25 \times 1.5 = 0.375\ m$$

$$Spacing\ between\ driven\ element\ and\ first\ director = 0.31\ \lambda = 0.31 \times 1.5 = 0.465\ m$$

$$Spacing\ between\ two\ directors = 0.31\ \lambda = 0.31 \times 1.5 = 0.465\ m$$

$$Length\ of\ the\ array = 1.5\ \lambda = 1.5 \times 1.5 = 2.25\ m$$

7. **Calculate the directivity of 20 turn helix having α = 12^0, circumference equal to one wavelength**.

Sol:

Given data:

No.of turns (N) = 20
Pitch angel (α) = 12^0
Circumference (C) = 1 λ

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

$$S = C.\tan\alpha = \lambda\ \tan(12^0) = 0.2126\ \lambda$$

The directivity of helical antenna is given by

$$D = \frac{15\ N\ S\ C^2}{\lambda^3} = \frac{15 \times 20 \times 0.2126\lambda \times \lambda^2}{\lambda^3} = \frac{300 \times 0.2126\lambda^3}{\lambda^3} = 63.78$$

$$D = 10\ \log(63.78) = 18\ dB$$

8. Design a helical antenna to produce a circularly polarized waves for the following parameters of the helix

Circumference of helix = 2λ
No,of turns = 15

Sol:

Given data:

Circumference of helix(C) = 2λ
No,of turns (N) = 15
Axial Ratio (AR) = 1 for circular polarization

$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$
$$2S\lambda = C^2$$
$$S = \frac{C^2}{2\lambda} = \frac{(2\lambda)^2}{2\lambda} = 2\lambda$$
$$C = \pi D$$
$$D = \frac{C}{\pi} = \frac{2\lambda}{\pi} = 0.636\ \lambda$$
$$Length\ of\ the\ helix\ (A) = N.S = 15 \times 2\lambda = 30\lambda$$
$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(2\lambda)^2 + (2\lambda)^2} = 2.828\ \lambda$$

9. Design a helical antenna for the frequency of 320 MHz to produce circularly polarized waves for the following parameters of helix

Diameter of helix = 0.56 m
No.of turns = 20

Sol:

Given data:

Frequency (f) = 320 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{320 \times 10^6} = 0.9375\ m$$

Diameter of helix = 0.56 m
No.of turns = 20
Axial Ratio (AR) = 1 for circular polarization

$$C = \pi\ D = \pi \times 0.56 = 1.76\ m$$
$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$
$$2S\lambda = C^2$$
$$S = \frac{C^2}{2\lambda} = \frac{(1.76)^2}{2 \times 0.9375} = 1.65\ m$$
$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{1.65}{1.76}\right) = 43.15^0$$
$$Length\ of\ the\ helix\ (A) = N.S = 20 \times 1.65 = 33\ m$$
$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(1.65)^2 + (1.76)^2} = 2.412\ m$$

10. Design helical antenna to produce the elliptically polarized waves for the following parameters of helix

Axial Ratio = 1.5
No.of turns = 18
Diameter of the helix = 0.5 m
Frequency = 10 MHz.

Sol:

Given data:

Axial Ratio = 1.5
No.of turns = 18
Diameter of the helix = 0.5 m
Frequency = 10 MHz.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^6} = 30\ m$$

$$C = \pi\, D = \pi \times 0.5 = 1.57\ m$$

$$AR = 1.5 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

$$S = \frac{1.5 \times C^2}{2\lambda} = \frac{1.5(1.57)^2}{2 \times 30} = 0.061\ m$$

$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(0.061)^2 + (1.57)^2} = 1.57\ m$$

$$Length\ of\ the\ helix\ (A) = N.S = 18 \times 0.061 = 1.098\ m$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{0.061}{1.57}\right) = 2.22^0$$

11. Find the values of D, S, α, L of helical antenna if the frequency is 312 MHz, circumference is 1.72 m and length of the helix is 35 m having 20 turns.

Sol:

Given data:

Frequency (f) = 312 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{312 \times 10^6} = 0.96\ m$$

Circumference of helix(C) = 1.72 m

Length of helix (A) = 35 m

No.of turns (N) = 20

$$C = \pi D$$

$$D = \frac{C}{\pi} = \frac{1.72}{\pi} = 0.547\ m$$

$$A = NS$$

$$S = \frac{A}{N} = \frac{35}{20} = 1.75\ m$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{1.75}{1.72}\right) = 45.5^0$$

$$Length\ of\ each\ turn\ (L) = \sqrt{S^2 + C^2} = \sqrt{(1.75)^2 + (1.72)^2} = 2.453\ m$$

12. Find the values of C, D, S, A, L of helical antenna if the frequency is 1.5 GHz, pitch angel $\alpha = 44.3^0$ and no.of turns is 25 with AR = 1

Sol:

Given data:

Frequency (f) = 1.5 GHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2\ m$$

Pitch angel (α) = 44.3^0

No.of turns (N) = 25

Axial Ratio (AR) = 1

$$AR = 1 = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

$$2S\lambda = C^2$$

$$S = \frac{C^2}{2\lambda}$$

$$\alpha = 44.3^0 = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{C^2}{2\lambda\, C}\right) = \tan^{-1}\left(\frac{C}{2\lambda}\right)$$

$$44.3 = \tan^{-1}\left(\frac{C}{2 \times 0.2}\right)$$

$$C = 0.4 \tan(44.3) = 0.39$$

$$D = \frac{C}{\pi} = \frac{0.39}{\pi} = 0.12\ m$$

$$S = \frac{C^2}{2\lambda} = \frac{(0.39)^2}{2 \times 0.2} = 0.38\ m$$

$$A = N.S = 25 \times 0.38 = 9.5\ m$$

$$L = \sqrt{S^2 + C^2} = \sqrt{(0.38)^2 + (0.39)^2} = 0.54\ m$$

UNIT - III (Aperture antennas and Lens antennas)

HORN ANTENNAS

Types:

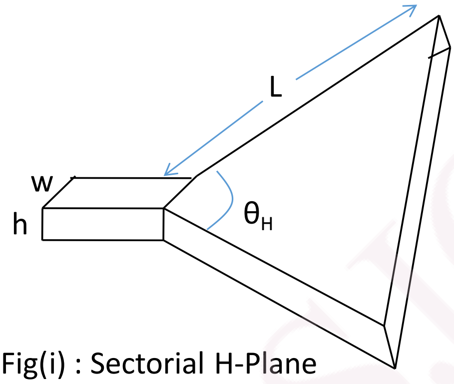
Horn antenna is a opened out or flared out waveguide. There are three advantages of flaring, such as

- (i) Impedance matching between waveguide and free space is obtained
- (ii) Diffraction problem will be eliminated.
- (iii) The EM waves can easily convert from guiding media (waveguide) into unguiding media (free space).

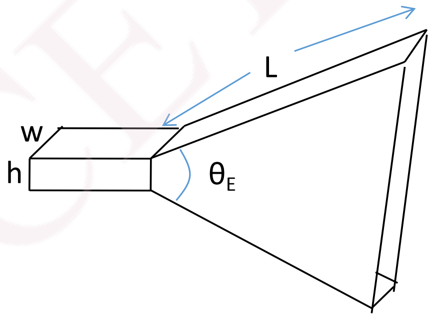
The types of horn antennas are given by

- (i) Sectorial H-Plane horn antenna
- (ii) Sectorial E-Plane horn antenna
- (iii) Pyramidal horn antenna
- (iv) Exponentially tapered pyramidal horn antenna
- (v) Conical horn antenna
- (vi) Exponentially tapered conical horn antenna

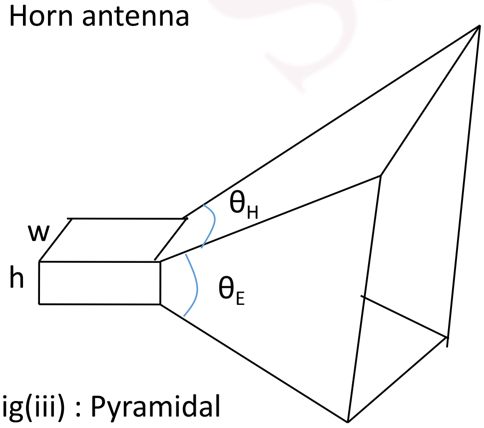
The structures of all the above antennas are shown in the figure below.\



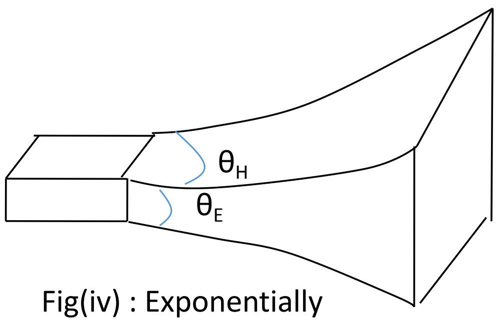
Fig(i) : Sectorial H-Plane Horn antenna



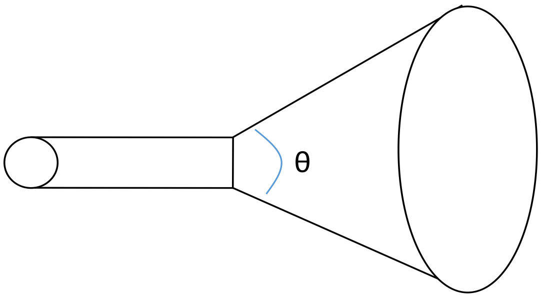
Fig(ii) : Sectorial E-Plane Horn antenna



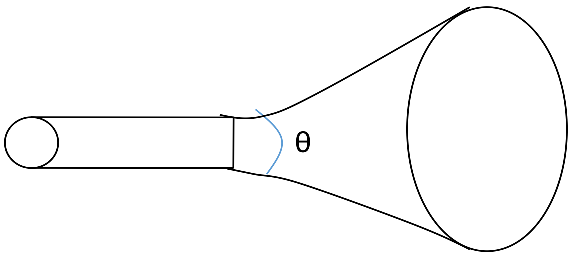
Fig(iii) : Pyramidal Horn antenna



Fig(iv) : Exponentially tapered Horn antenna



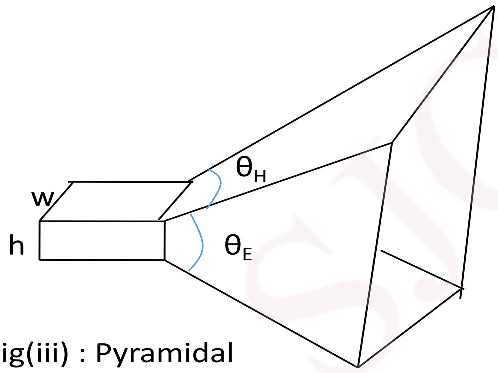
Fig(v) : Conical horn antenna



Fig(vi) : Exponentially Conical horn antenna

Fermat's Principle and Optimum horns:

The statement of Fermat's principle is "equality of physical path lengths or equality of electrical path lengths". To get the maximum filed strength at the receiving point, the Fermat's principle must be satisfied. In case of horn antenna, the Fermat's principle is not satisfied exactly because there is deviation (δ) between the waves at the center and waves at the edges but satisfied with some relaxation. The pyramidal horn antenna and its cross section is shown in figure below.



Fig(iii) : Pyramidal Horn antenna

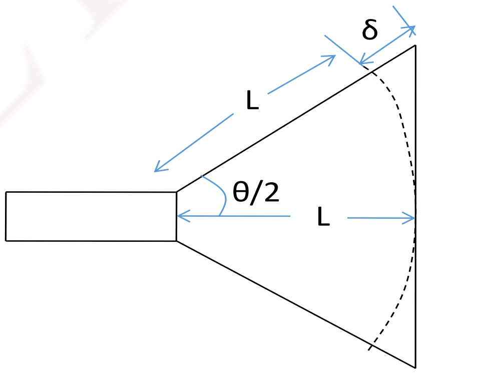
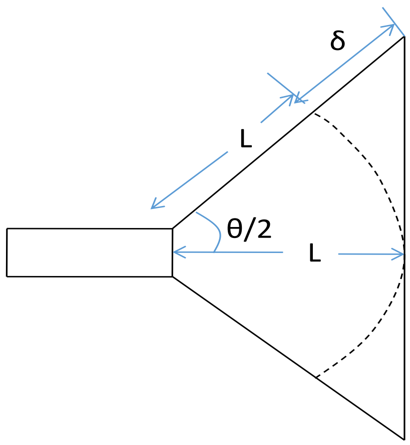
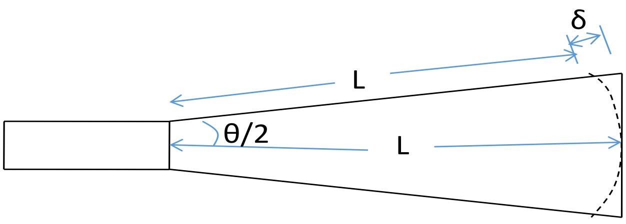


Fig : Cross section

The relaxation is such that, the deviation (δ_E) along the E-Plane should not be greater than 0.25λ and the deviation (δ_H) along H-Plane should not be greater than 0.4λ.
In case of horn antenna, the parameters flare angel (θ) and length (L) should be properly selected otherwise certain problems will arise. When the flare angle (θ) is large and length (L) is small, then there will be more deviation (δ), but the advantage is the construction is easy. When the length (L) is large and flare angle (θ) is small, then it is difficult to construct such large length but the advantage is less deviation (δ). These two situations are shown in the figures below



Fig(i) : Cross section of Pyramidal Horn antenna when θ is large and L is small



Fig(ii) : Cross section of Pyramidal Horn antenna when θ is small and L is large

Therefore we need to compromise between these two satiations. That is we need to select optimum values for flare angle and length. The equations for the optimum values of deviation (δ_o) and length (L) is given by

$$\delta_o = \frac{L}{cos\left(\frac{\theta}{2}\right)} - L$$

$$L = \frac{\delta_o cos\left(\frac{\theta}{2}\right)}{1 - cos\left(\frac{\theta}{2}\right)}$$

<u>Design considerations of Pyramidal Horns:</u>

Pyramidal horn antenna is a combination of sectorial E-plane horn and sectorial H-plane horn. Therefore designing of pyramidal horn is nothing but designing of sectorial E-plane and sectorial H-plane horan antennas. Let us consider the designing of these two antennas.

Sectorial E-Plane Horn antenna: The cross section of sectorial E-polane horn antenna is shown in the figure below.

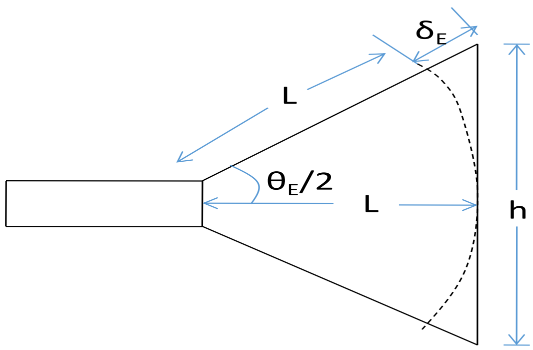


Fig : Cross section of Sectorial E-Plane Horn antenna

From the above figure,

$$\cos\left(\frac{\theta_E}{2}\right) = \frac{L}{L+\delta_E}$$

$$\theta_E = 2\cos^{-1}\left(\frac{L}{L+\delta_E}\right)$$

Or

$$\sin\left(\frac{\theta_E}{2}\right) = \frac{h/2}{L+\delta_E}$$

$$\theta_E = 2\,sin^{-1}\left(\frac{h}{2(L+\delta_E)}\right)$$

Or

$$\tan\left(\frac{\theta_E}{2}\right) = \frac{h/2}{L}$$

$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right)$$

From figure,

$$(L+\delta_E)^2 = L^2 + \left(\frac{h}{2}\right)^2$$

$$L^2 + {\delta_E}^2 + 2L\delta_E = L^2 + \frac{h^2}{4}$$

$$L = \frac{h^2}{8\delta_E}$$

Sectorial H-Plane Horn antenna: The cross section of sectorial H-polane horn antenna is shown in the figure below.

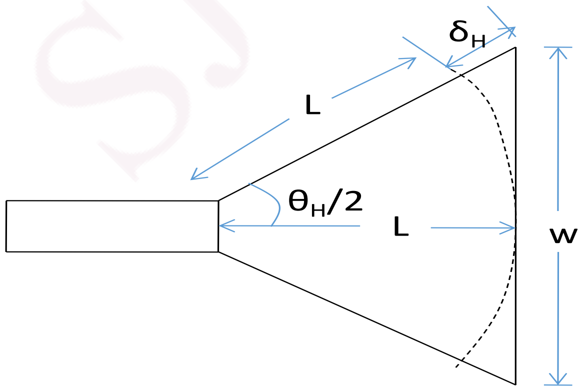


Fig : Cross section of Sectorial H-Plane Horn antenna

From the above figure,

$$\cos\left(\frac{\theta_H}{2}\right) = \frac{L}{L+\delta_H}$$

$$\theta_H = 2\cos^{-1}\left(\frac{L}{L+\delta_H}\right)$$

Or

$$\sin\left(\frac{\theta_H}{2}\right) = \frac{w/2}{L+\delta_H}$$

$$\theta_H = 2\,sin^{-1}\left(\frac{w}{2(L+\delta_H)}\right)$$

Or

$$\tan\left(\frac{\theta_H}{2}\right) = \frac{w/2}{L}$$

$$\theta_H = 2\tan^{-1}\left(\frac{w}{2L}\right)$$

From figure,

$$(L+\delta_H)^2 = L^2 + \left(\frac{w}{2}\right)^2$$

$$L^2 + {\delta_H}^2 + 2L\delta_H = L^2 + \frac{w^2}{4}$$

$$L = \frac{w^2}{8\delta_H}$$

The Half Power Beam Width (HPBW) along E-Plane and H-plane directions are given by

$$\theta_{E(HPBW)} = \frac{56\lambda}{h}\ degree$$

$$\theta_{H(HPBW)} = \frac{67\lambda}{w}\ degree$$

The directivity of pyramidal horn antenna is given by

$$D = \frac{7.5\ h.w}{\lambda^2} = \frac{7.5\ A}{\lambda^2}$$

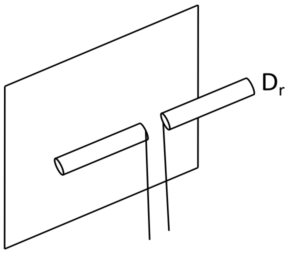
The power gain (G_P) of pyramidal horn antenna is given by

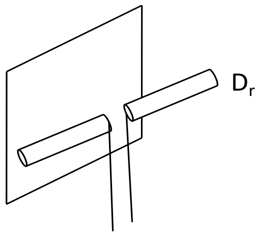
$$G_P = \frac{4.5\ h.w}{\lambda^2} = \frac{4.5\ A}{\lambda^2}$$

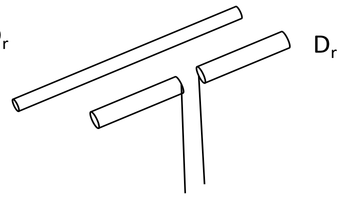
REFLECTOR ANTENNAS:

Introduction:

(i) Reflectors are made from good conductor.
(ii) Reflectors are used to modify the radiation pattern of the antenna.
(iii) The reflectors are also used to eliminate the back lobe radiation
(iv) The reflectors are used to convert the bidirectional radiation pattern in to unidirectional radiation pattern.
(v) There are various types of reflectors such a flat sheet reflector, thin linear reflector, Corner reflector, parabolic reflector, elliptical reflector, circular reflector, etc.
(vi) The structure of various types of reflectors are shown in the figure below



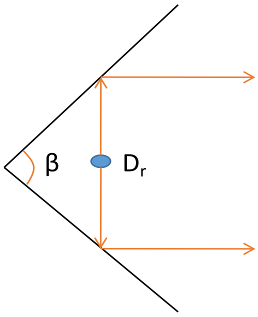


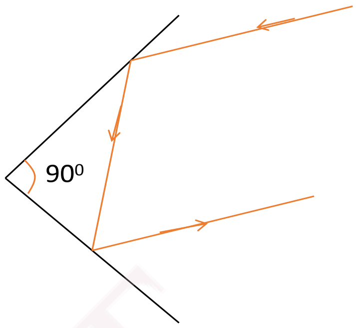


(a) Small flat sheet reflector

(a) Thin linear reflector

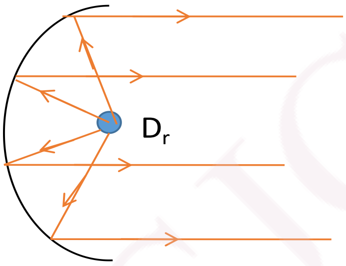
(a) Large flat sheet reflector

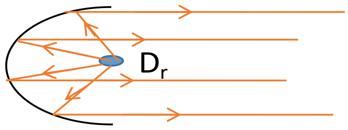




(d) Active corner reflector

(e) Passive corner or retro reflector





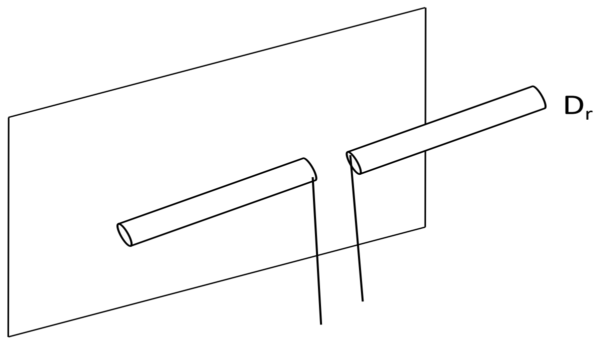
(g) Elliptical reflector

(f) Parabolic reflector

FLAT SHEET or FLAT PLATE REFLECTORS:

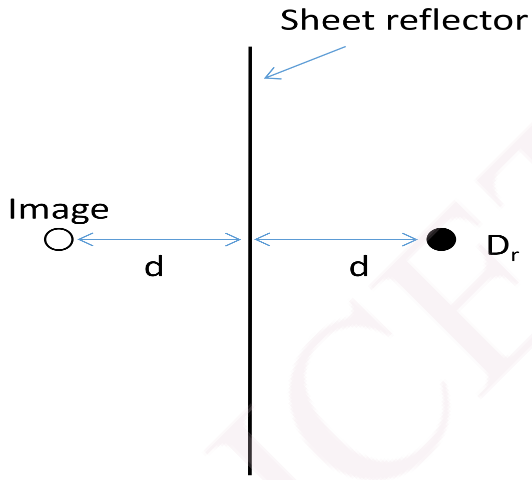
(i) When a flat metal sheet is used at backside of the antenna (driven element) to reflect back the signal then it is known as flat sheet reflector.

(ii) The structure of flat sheet reflector antenna is shown in figure below.



flat sheet reflector

(iii) The reflector antennas will be analyzed by using the method of images.



(iv) Flat sheet reflector antenna can be analyzed by imagining as a combination of two antennas separated by certain distance (d).

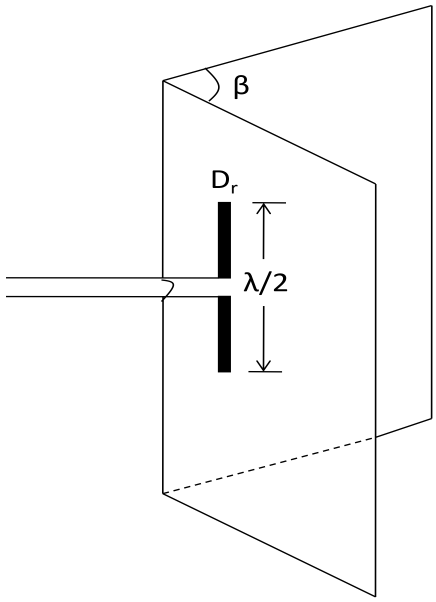
(v) A large flat sheet reflector can convert bidirectional pattern in to unidirectional pattern.

(vi) The distance(d) between the reflector and the driven element will decide the directional properties of antenna

<u>CORNER REFLECTORS:</u>

(i) When two flat metal sheets are meeting at an angle or corner, then it is known as corner reflector antenna.

(ii) The basic structure of corner reflector antenna is shown in the figure below.



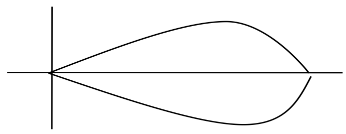


Fig: Radiation pattern

Fig: Corner reflector antenna

(iii) There are two types of corner reflector antennas such as active corner reflector antenna and passive corner reflector antenna.

(iv) The active corner reflector contains the driven element, where as passive corner reflector do not contain the driven element.

(v) The corner angle β is given by

$$\beta = \frac{180^0}{n}$$

Where n = an integer = 1,2,3,……

(vi) When corner angle is equal to 180^0(when n = 1), then it is called as flat sheet reflector.

(vii) When the corner angle is equal to 90^0 (when n = 2), then it is known as square corner reflector.

<u>Square corner reflector:</u>

- The structure of square corner reflector is shown in the figure below

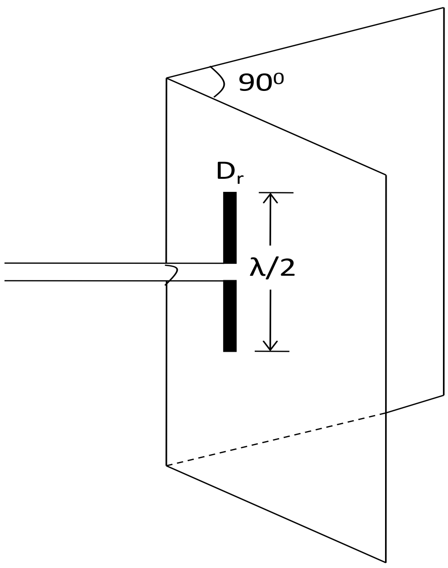


Fig: Square Corner reflector antenna

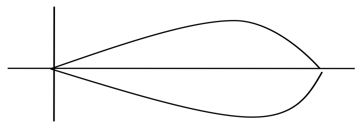
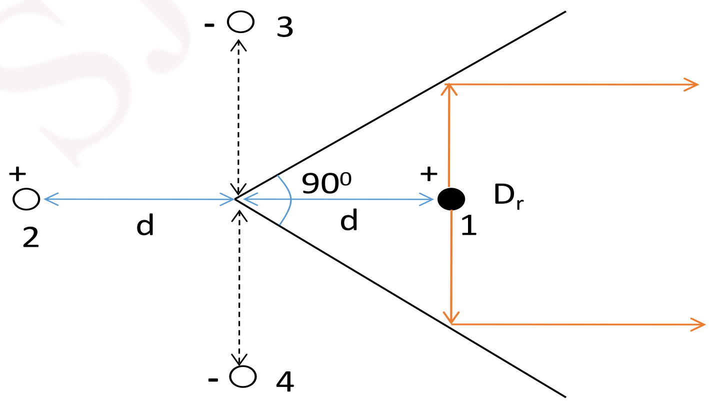


Fig: Radiation pattern

- The square corner reflector antenna contains corner reflector and driven element.
- The driven element will be preferably half wave dipole.
- A square corner reflector without driven element is called passive reflector or retro reflector.
- As per the method of images, square corner reflector can be imagined as combination of four antennas (three images and one driven element) as shown in figure below.



- It is a combination of two two-element arrays.
- The gain of square corner reflector is derived as follows:

The filed pattern $E_\varphi(\theta)$ in the horizontal plane at a large distance r from the antenna is given by

$$E_\varphi(\theta) = k^{\prime} I_1[cos(\beta d cos\theta) - cos(\beta d sin\theta)] \qquad -(1)$$

Where $k^{\prime}$ is the constant involving distance r

I_1 is the current in each element, $\beta = 2\pi/\lambda$ called phase constant, d is the distance between the driven element and the corner.

The terminal voltage at the centre of the driven element (half wave dipole) is given by

$$V_1 = I_1Z_{11} + I_2Z_{12} - I_1Z_{13} - 1Z_{14}$$

But $Z_{13} = Z_{14}$,

Then
$$V_1 = I_1(Z_{11} + Z_{12} - 2Z_{14})$$

Where Z_{11} is the self impedance of driven element, Z_{12} is the mutual impedance between the element 1 and 2, Z_{14} is the mutual impedance between element 1 and 4. The above equation can also be written as

$$V_1 = I_1(R_{11} + R_{12} - 2R_{14})$$

Now if P be the power supplied to the driven antenna, then

$$P = I_1^2R$$

$$I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \qquad -(2)$$

By substituting equation (2) in equation (1),

$$E_\varphi(\theta) = k' \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \times [cos(\beta dcos\theta) - cos(\beta dsin\theta)] \qquad -(3)$$

When the reflector is removed then equation (3) becomes

$$E_\varphi(\theta)_{\lambda/2} = k' \sqrt{\frac{P}{R_{11}}} \qquad -(4)$$

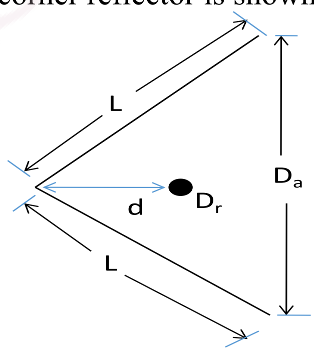
Above equation represents the electric field strength of half wave dipole which will be used as the reference for obtaining the gain of the corner reflector antenna. The ratio between the equations (3) and (4) gives the gain of the square corner reflector antenna and is given by

$$G = \frac{E_\varphi(\theta)}{E_\varphi(\theta)_{\lambda/2}} = \sqrt{\frac{R_{11}}{R_{11} + R_{12} - 2R_{14}}} \times [cos(\beta dcos\theta) - cos(\beta dsin\theta)]$$

In above equation the term $cos(\beta dcos\theta) - cos(\beta dsin\theta)$ is called as pattern factor and the term $\sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}}$ is called as coupling factor.

Design consideration of square corner reflector:

The dimensions of the square corner reflector is shown in the figure below

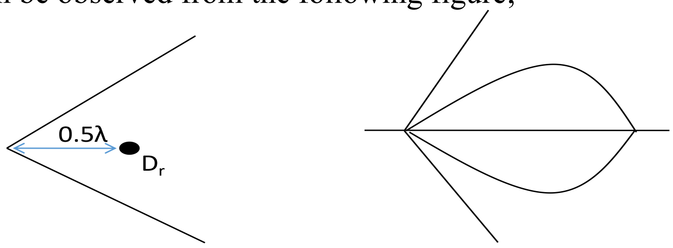


From the above figure the following formulae can be deduced

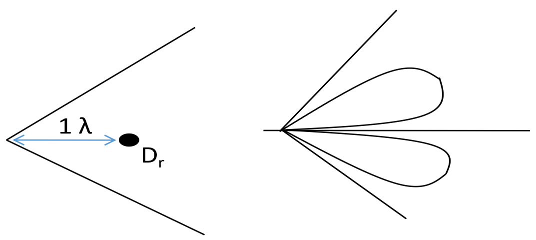
$$d = \frac{L}{2}$$
$$L = 2d$$
$$D_a = \sqrt{L^2 + L^2} = L\sqrt{2} = 1.414\, L$$
$$D_a = 1.414\, L = 1.414\,(2d) = 2.828\, d$$

Effect of spacing between the driven element and corner of the reflector (d) on the pattern:

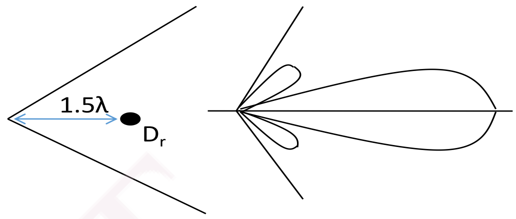
When the spacing (d) between the driven element and corner of the reflector is varied, the pattern characteristics such as beam width, gain, minor lobes, etc will be affected. This effect can be observed from the following figure;



Fig(a): When d = 0.5 λ



Fig(b): When d = 1 λ



Fig(c): When d = 1.5 λ

Retro or passive Square corner reflector:

- The square corner reflector without any driven element is known as retro reflector of passive square corner reflector.
- The structure of passive square corner reflector is shown in the following figure

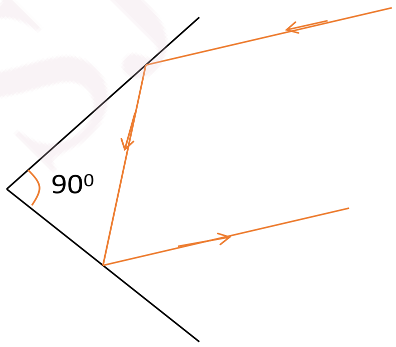


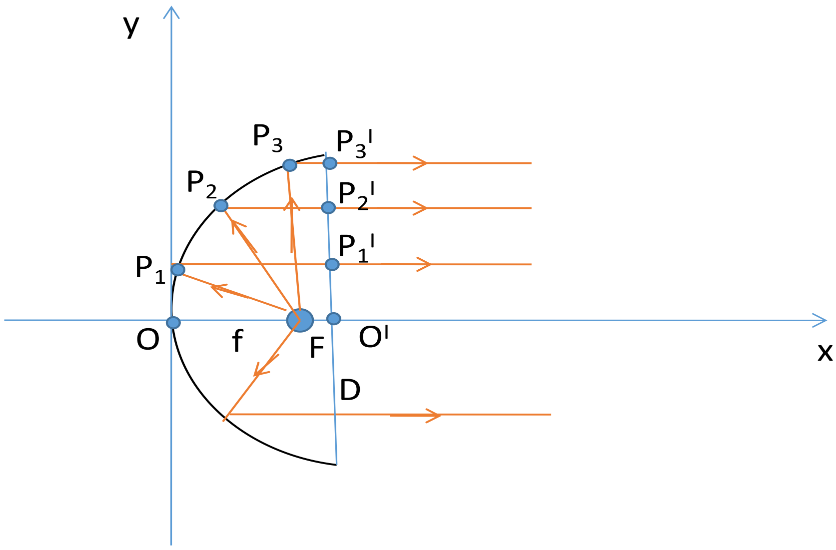
Fig:Passive corner or retro reflector

- The corner angel of retro reflector is 90^0.
- The retro reflector will be used as the target for the radar system.

PARABOLA REFLECTORS:

Geometry:

- The geometry or constructional features of parabola reflector is shown in the figure below.



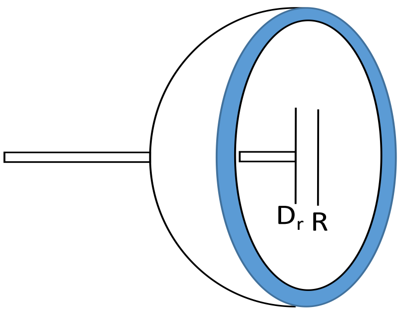
- In above figure, OF represents Focal length (f), F is called Focus, O is called Vertex, D is called Directrix or aperture size and OO^1 is called axis of parabola.
- Parabola is defined as the locus of a point which moves in such a way that, its distance from a fixed distance called focus plus its distance from a straight line called directrix is constant i.e.

$$FP_1 + P_1P_1^{/} = FP_2 + P_2P_2^{/} = FP_3 + P_3P_3^{/} = constant$$

- The equation of parabola in terms of its coordinates is given by

$$y^2 = 4fx$$

- The ratio of focal length (f) to Aperture size (D) is known as f over D ratio or simple f/D.
- When a parabola is rotated about its axis OO^1, then it is known as paraboloidal reflector.
- The parabola is a two dimensional and paraboloid is three dimensional.
- The structure of paraboloidal reflector is shown in the figure below.



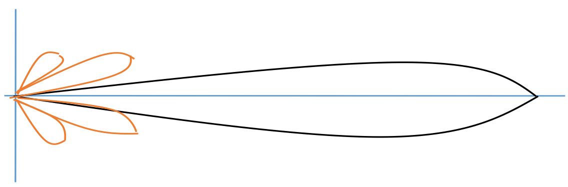


Fig: Radiation Pattern

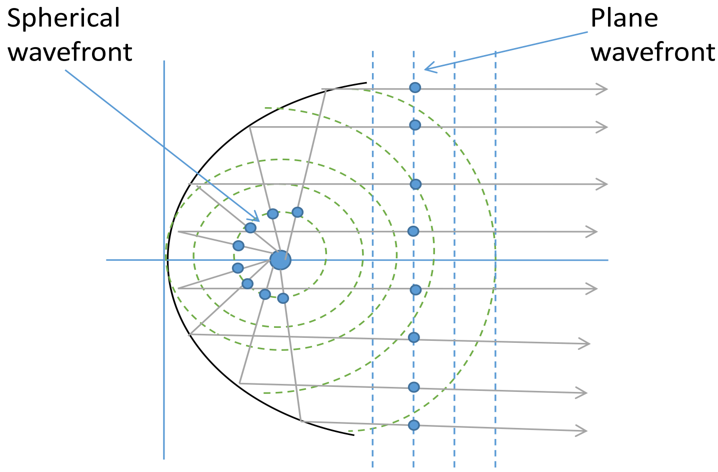
Fig: Paraboloidal reflector

- The equation of the paraboloidal reflector is given by

$$y^2 + z^2 = 4fx$$

Pattern characteristics:

- The parabolic reflector converts spherical wavefront into plane wavefront as shown in the figurer below.



- It will convert bidirectional radiation pattern in to unidirectional pattern.
- The radiation pattern characteristics or directional characteristics of reflector antenna depend upon the f over D ratio.
- When f/D ratio is small (as shown in figure below), then all the waves radiated by the driven element will be reflected by the reflector, but uniform illumination of reflector by the source (driven element) is not possible.

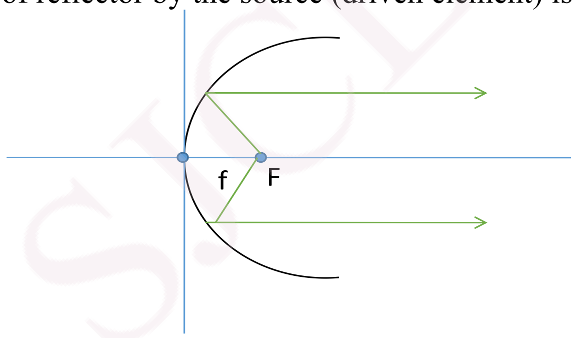


Fig: f/D < 1/4

- When f/D ratio is large as shown in the figure below, some of the waves will escape without reflection by the reflector and will constitute a spill over.

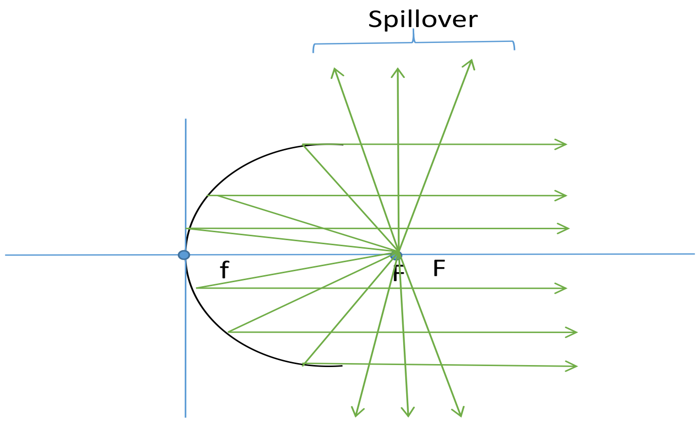


Fig: f/D > 1/4

- Practically f/D ratio will be selected in between small and large, typically f/D =1/4 as shown in the figure below.

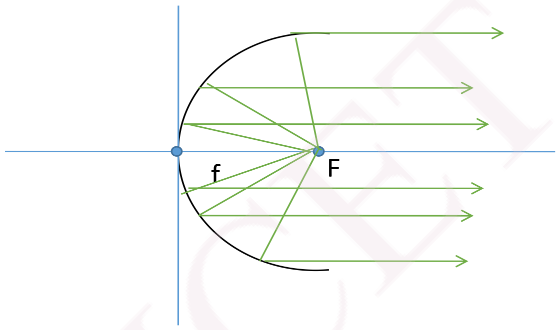


Fig: f/D = 1/4

- f over D ratio of parabolic reflector antenna is given by

$$\frac{f}{D} = \frac{1}{4} cot\left(\frac{\theta}{2}\right) = 0.25 \cot{(\theta/2)}$$

- The HPBW(Half Power Beam Width) of parabolic reflector with circular aperture is given by

$$HPBW = \frac{58\lambda}{D} \ degree$$

- The BWFN (Beam Width between First Nulls) of a parabolic reflector with circular aperture is given by

$$BWFN = \frac{140\lambda}{D} \ degree$$

- The directivity of a parabolic reflector with uniform illumination is given by

$$Directivity = 9.87\left(\frac{D}{\lambda}\right)^2$$

Where D represents the diameter of the circular aperture.

Feed Methods:

- Feed is nothing but a driven element. An ideal feed is one which illuminates the reflector uniformly.

- There are different types of methods for feeding the parabolic reflector antenna such as Dipole end fire feed, Horn feed, Cassegrain feed and Offset feed.
- The geometry of dipole end fire feed is shown in the following figure.

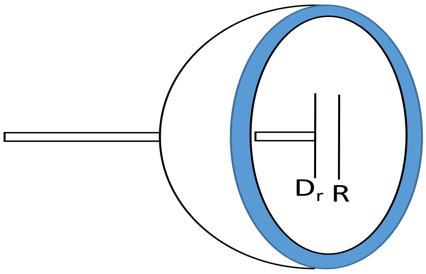


Fig: Dipole end fire feed

- In dipole end fire feed, the driven element is a two-element end fire array.
- With dipole feed, the driven element should be located at focus only.
- The most common feed method for paraboloid reflector is horn feed which is shown in the figure below.

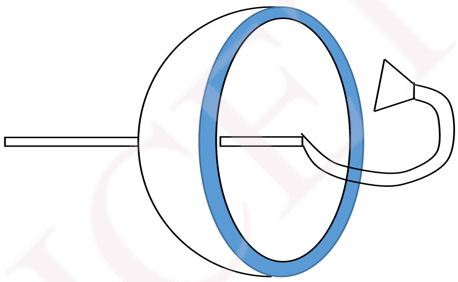


Fig: Horn feed

- In horn feed method, the horn antenna is used as the driven element.
- The horn antenna will be located at the focus such that, it can illuminate the reflector uniformly.
- The horn antenna will be located at the focus with the help of waveguide support.
- The drawback of horn feed is, the waveguide support will obstruct the waves which will affect the radiation pattern.
- In dipole feed and horn feed, the spillover will be present.
- To avoid the drawbacks of dipole feed and horn feed the cassegrain feed will be used
- The geometry of cassegrain feed is shown in the figure below.

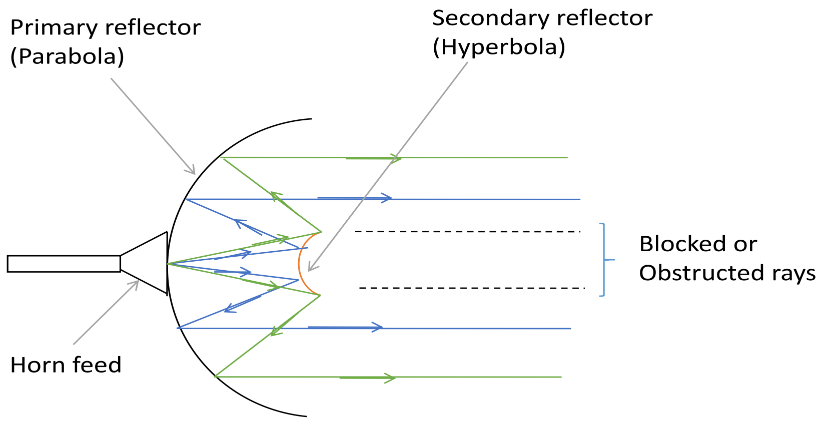


Fig: Geometry of cassegrain feed

- In cassegrain feeding, one more reflector called secondary reflector (preferably hyperbola) in addition to the primary reflector (Parabola) will be used.
- In cassegrain feed, the driven element can be located at our convenient location (preferably it will be located at vertex).
- The major advantages of cassegrain feed over the other methods are
  (i) Reduction in spill over and minor lobe radiation
  (ii) Ability to get an equivalent focal length much greater than the physical length.
  (iii) It allows us to place the feed at a convenient location
- The major drawback of cassegrain feed is, aperture blocking i.e. some of the area (aperture) of parabola is blocked by the secondary reflector hyperbola.
- To eliminate the drawback of cassegrain feeding, an offset feeding will be used.
- The structure of offset feed method is shown in figure below

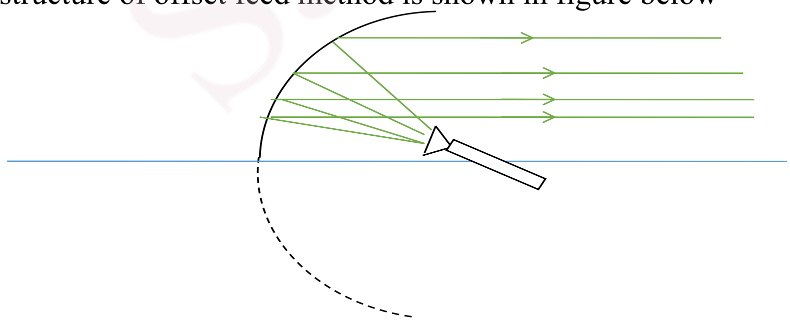


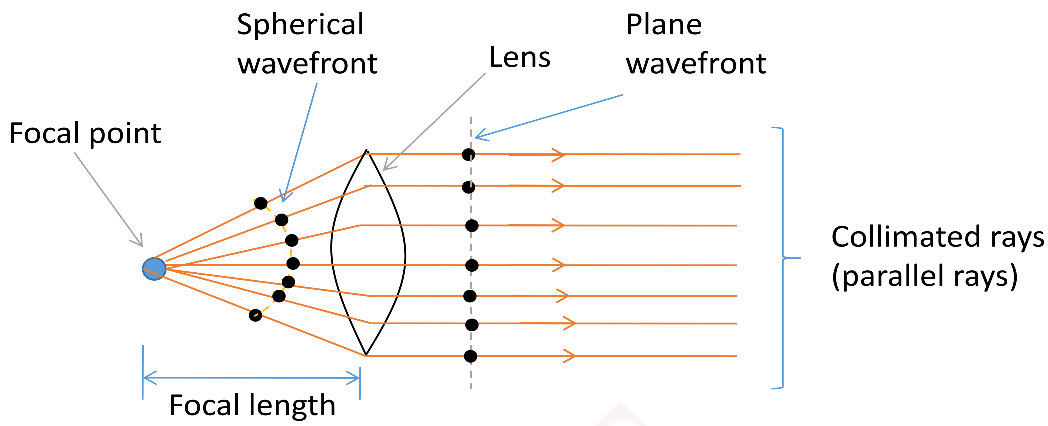
Fig: Offset feed

- In offset feeding, the horn antenna (driven element) will be arranged to illuminate only half of the parabola.

LENS ANTENNAS

Basic Principle:

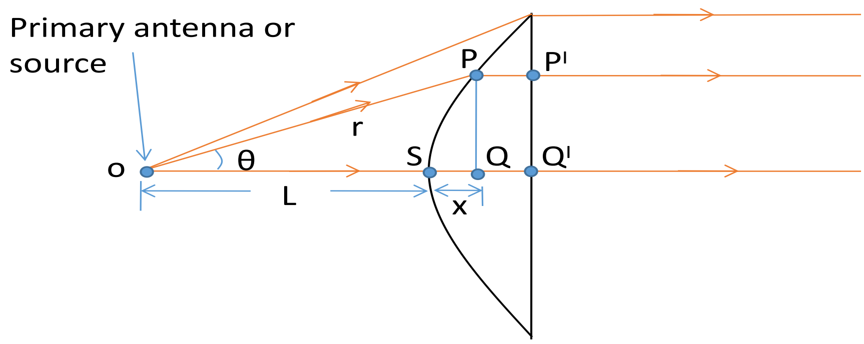
- Lens antennas are made from the lenses, preferably with concave and convex lenses.
- The collimating action of lens antenna can be understood from the following figure.



- Assuming the source or driven element at focal point at a distance of focal length, along the lens axis, it is seen that collimated or parallel rays are obtained on the right side of the lens.
- An optical lens operates by virtue of having a refractive index more than the unity.
- The principle of "equality of path length" called Fermat's principle will be applicable to then lens antennas.
- Lens medium can be used either to increase the speed of the waves or to decrease the speed of the waves.

Geometry of Non-metallic Dielectric Lenses:

- Basically there are two types of lens antennas such as Dielectric lens (or H-plane metal plate lens or Delay lens) and E-plane metal plate lens.
- The dielectric lens antennas are sub divided in to two types such as Non-metallic dielectric type and Metallic or Artificial dielectric type of lens.
- Non-metallic dielectric lens antenna will be made from Plano-concave lens.
- The geometry or constructional features of non-metallic dielectric lens antenna is shown in the figure below.



- The non-metallic dielectric lens will reduce the velocity of the waves such that equality of path length will be satisfied.
- The equation for the contour of the lens will be obtained as follows:
  From figure,
  OP + PP' = OS + SQ' = OS + SQ + QQ'   but PP' = QQ'
  Then   OP = OS + SQ
$$\frac{r}{c} = \frac{L}{c} + \frac{x}{v}$$
$$r = L + \left(\frac{c}{v}\right)x = L + \mu x$$
  But
$$x = r\cos\theta - L$$
$$r = L + \mu(r\cos\theta - L)$$
$$r = \frac{L(\mu - 1)}{\mu\cos\theta - 1}$$
  Where $\mu = \frac{c}{v}$ is called as refractive index of the lens medium.
  The above equation represents the equation of hyperbola whose focal length is L and radius of curvature (R). Where R = L(μ-1).

Zoning:

- The process of reducing the size by removing certain portions of the lens antenna is called lens zoning.
- At low frequencies, the size of the lens antenna becomes bulky, because the size of the lens is inversely proportional to the frequency.
- The structures of zoned or stepped lens are shown in the figure below.

Fig: Zoned or stepped lens dielectric lenses

- The thickness of stepped or zoned lens is given by
$$t = \frac{\lambda}{\mu - 1}$$
- The zoned lens antenna is depends upon frequency. The zoned lens antennas can be used at low frequencies also.

Tolerances:

- Two important parameters to be considered while designing the lens antenna are thickness of the lens and refractive index of the lens medium.
- If there is any deviation in thickness from the ideal contour and any variations in the refractive index of the lens, there will be different path length of the

waves passing through the lens. As a result equality of path length will not be satisfied.

- Therefore, there must be allowable variation to both thickness and refractive index.ie. Tolerances on thickness and refractive index should be considered.
- Tolerance on thickness(Δt) is given by

$$\Delta t = \frac{\lambda_0}{32(\mu - 1)} = \frac{0.03\lambda_0}{\mu - 1}$$

- The tolerance on the index of refraction or refractive index (μ) is given by

$$\Delta \mu = \frac{0.03}{t_\lambda}$$

Where t_λ is the thickness of lens in free-space wavelength.

- Following table gives the tolerances on thickness and refractive index of various lens antennas:

S.No	Type of Antenna	Type of tolerance	Amount of tolerance(rms)
1	Dielectric lens(unzoned)	Thickness	$\frac{0.03\lambda_0}{\mu - 1}$
		Index of refraction	$\frac{3}{\mu t_\lambda}\%$
2	Dielectric lens(zoned)	Thickness	3%
		Index of refraction	$\frac{3(\mu - 1)}{\mu}\%$
3	E-plane metal plate lens(unzoned)	Thickness	$\frac{0.03\lambda_0}{1 - \mu}$
		Plate spacing	$\frac{3\mu}{1 - \mu^2} t_\lambda\%$
4	E-plane metal plate lens(unzoned)	Thickness	3%
		Plate spacing	$\frac{3\mu}{1 + \mu}\%$

SLOT ANTENNA:

The antenna shown in figure 1, consisting of two resonant λ/4 stubs connected to a 2-wire transmission line, is an inefficient radiator. The long wires are closely spaced (w << λ) and carry currents of opposite phase so that their fields tend to cancel. The end wires carry currents in the same phase, but they are too short to radiate efficiently. Hence enormous currents may be required to radiate appreciable amount of power.

The antenna shown in figure 2, on the other hand, is a very efficient radiator. In this arrangement a λ/2 slot is cut in a flat metal sheet. Although the width of the slot is small (w << λ), the currents are not confined to the edges of the slot but spread out over the sheet. This is a simple type of slot antenna. Radiation occurs equally from both sides of the sheet. If the slot is horizontal, the radiation normal to the sheet is vertically polarized.

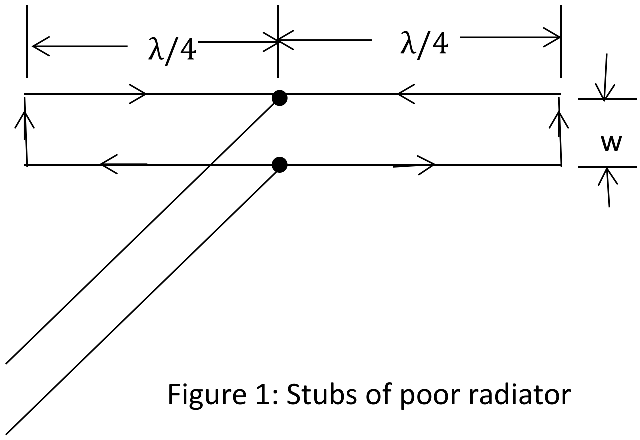


Figure 1: Stubs of poor radiator

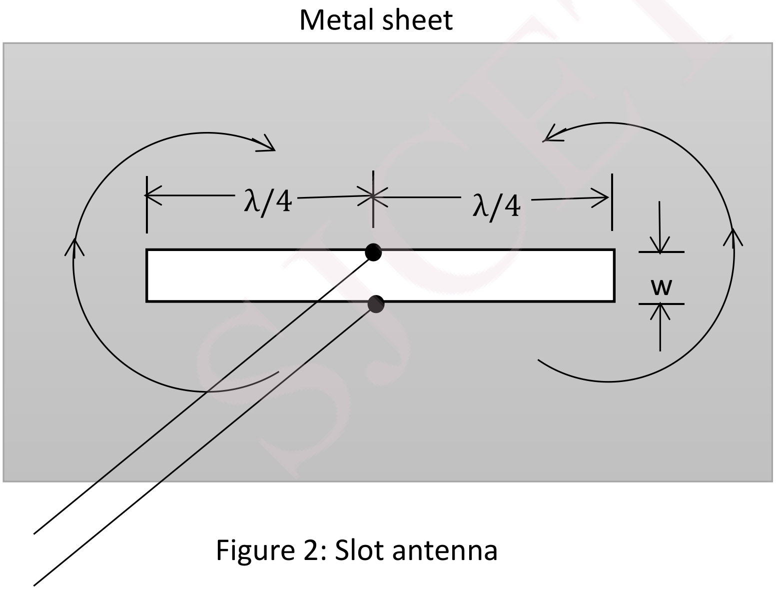


Figure 2: Slot antenna

A slot antenna may be conveniently energized with a coaxial transmission line as shown in figure 3. The outer conductor of the cable is bonded to the metal sheet. Since the terminal resistance at the center of a resonant λ/2 slot in a large sheet is about 500 Ω and the characteristic impedance of coaxial transmission line is usually much less, an off-center feed such as shown in figure 4 may be used to provide a better impedance match. For a 50 Ω coaxial cable the distance ‘s’ should be about λ/20. Slot antennas fed by a coaxial line in this manner are illustrated in figure 5 and figure 6. The radiation normal to the sheet with the horizontal slot(figure 5) is vertically polarized while radiation normal to the sheet with the vertical slot(figure 6) is horizontally polarized.

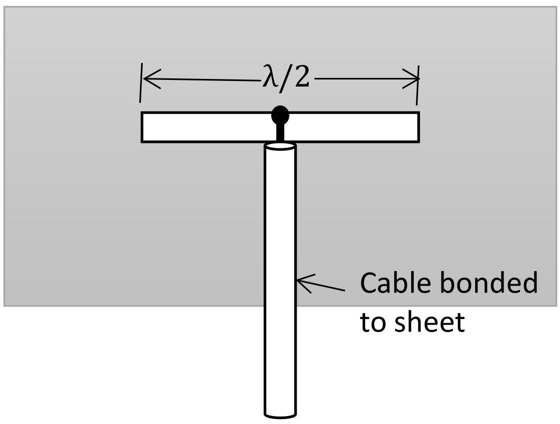


Figure 3: Slot antenna fed at center

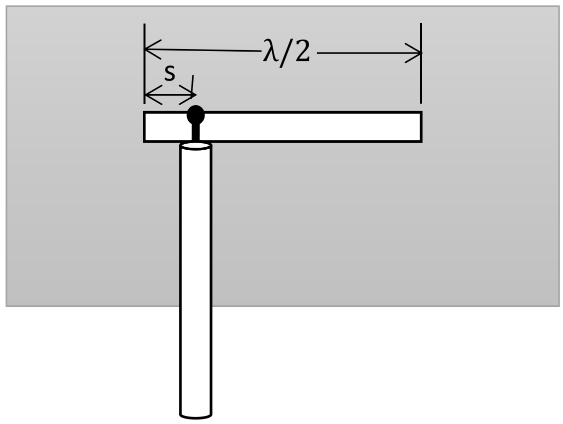


Figure 4: Slot antenna fed at off-center

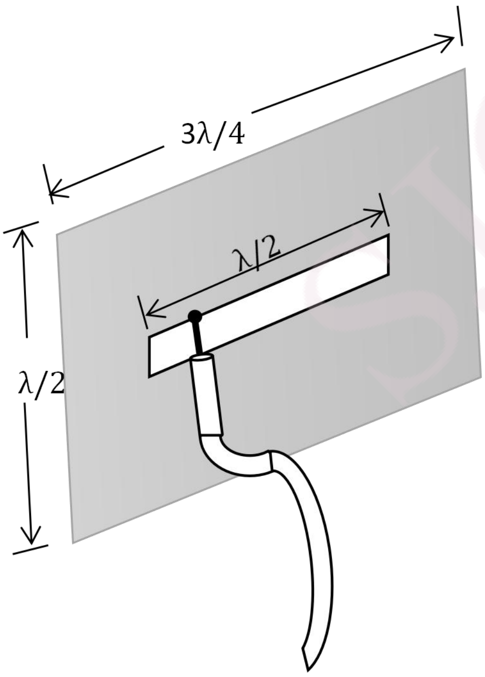


Figure 5: Vertically polarized slot antenna

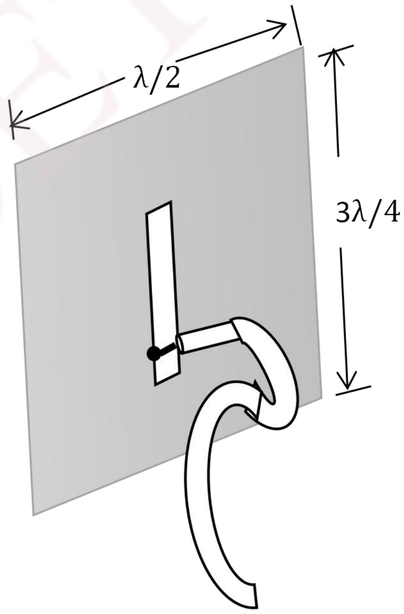


Figure 6: Horizontally polarized slot antenna

The radiation pattern of $\lambda/2$ slot in an infinite sheet is shown in figure 7. The infinite flat sheet is coincident with the xz plane and the long dimension of the slot is in the x direction.

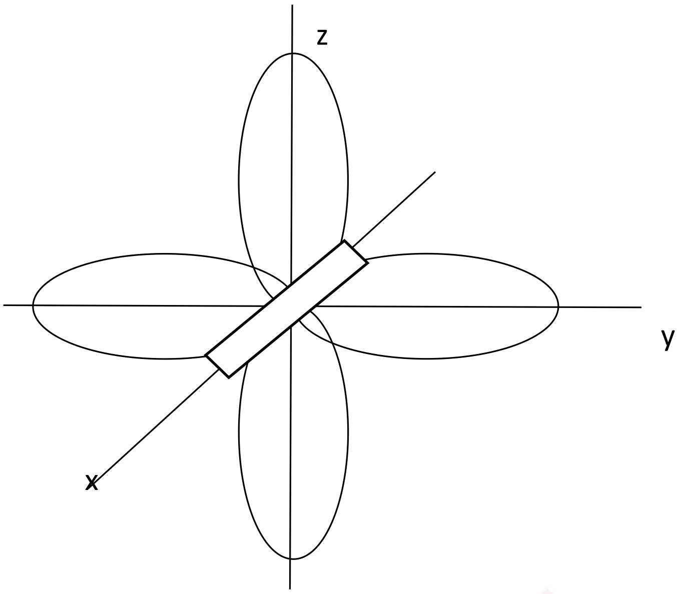


Figure 7: Radiation pattern of slot antenna

SOLVED PROBLEMS

1. Calculate the power gain of an optimum horn antenna approximately with a square aperture of 10λ on a side

Sol:

Give data:

Side of the horn antenna = 10 λ

The power gain of horn antenna is given by

$$G_P = \frac{4.5\,A}{\lambda^2} = \frac{4.5 \times 10\lambda \times 10\lambda}{\lambda^2} = 450$$

$$G_P = 10\log(450) = 16.53\;dB$$

2. Find out the length L, width W and flare angles $\boldsymbol{\theta_E}$ and $\boldsymbol{\theta_H}$ of a pyramidal horn antenna for which the mouth height h = 10λ. The horn is fed by a rectangular waveguide with TE_{10} mode.

Sol:

Given data:

$$Mouth\ height(h) = 10\lambda$$

Let

$$\theta_E = 0.25\,\lambda\ \ and\ \theta_H = 0.4\lambda$$

$$L = \frac{h^2}{8\delta_E} = \frac{(10\lambda)^2}{8 \times 0.25\lambda} = 50\lambda$$

$$L = \frac{w^2}{8\delta_H}$$

$$w = \sqrt{8\delta_H L} = \sqrt{8 \times 0.4\lambda \times 50\lambda} = 12.65\,\lambda$$

$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2 \times 50\lambda}\right) = 2 \times 5.71^0 = 11.42^0$$

$$\theta_E = 2\tan^{-1}\left(\frac{w}{2L}\right) = 2\tan^{-1}\left(\frac{12.65\lambda}{2 \times 50\lambda}\right) = 2 \times 7.2^0 = 14.4^0$$

3. (a) Determine the length L, H-plane aperture and flare angles $\boldsymbol{\theta_E}$ and $\boldsymbol{\theta_H}$ (in the E and H planes respectively) of a pyramidal horn for which E-plane aperture a_E = 10λ. The horn is fed by a rectangular waveguide with TE_{10} mode. Let δ = 0.2 λ in the E-plane and 0.375 λ in the H-plane (b) What are the beamwidths (c) What is the directivity

Sol:

Given data:

E-plane aperture (a_E = h) = 10λ

$$\delta_E = 0.2\lambda \ and \ \delta_H = 0.375\lambda$$

(a)

$$L = \frac{h^2}{8\delta_E} = \frac{(10\lambda)^2}{8 \times 0.2\lambda} = 62.5\lambda$$

$$L = \frac{w^2}{8\delta_H}$$

$$w = \sqrt{8\delta_H L} = \sqrt{8 \times 0.375\lambda \times 62.5\lambda} = 13.7\ \lambda$$

$$\theta_E = 2\tan^{-1}\left(\frac{h}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2 \times 62.5\lambda}\right) = 2 \times 4.57^0 = 9.1^0$$

$$\theta_E = 2\tan^{-1}\left(\frac{w}{2L}\right) = 2\tan^{-1}\left(\frac{13.7\lambda}{2 \times 62.5\lambda}\right) = 2 \times 6.25^0 = 12.5^0$$

(b)

$$\theta_{E(HPBW)} = \frac{56\lambda}{h}\ degree = \frac{56\lambda}{10\lambda} = 5.6^0$$

$$\theta_{H(HPBW)} = \frac{67\lambda}{w}\ degree = \frac{67\lambda}{13.7\ \lambda} = 4.9^0$$

(c)

Directivity

$$D = \frac{7.5\ h.w}{\lambda^2} = \frac{7.5 \times 10\lambda \times 13.7\ \lambda}{\lambda^2} = 1027.5$$

$$D = 10\log(1027.5) = 30.1\ dB$$

4. Find out the power gain in dB of a paraboloidal reflector of open mouth aperture 10λ

Ans:

Given

Mouth aperture (D) = 10λ

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{10\lambda}{\lambda}\right)^2 = 640$$

$$G_p = 10\log(640) = 28\ dB$$

5. Find out the beam width between first nulls and power gain of a 2 m paraboloid reflector operating at 6000 MHz.

Ans:

Given

Diameter (D) = 2 m

Frequency (f) = 6000 MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6000 \times 10^6} = 0.05\ m$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.05}{2} = 3.5\ degree$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{2}{0.05}\right)^2 = 10240$$

$$G_p = 10\log(10240) = 40\ dB$$

6. A parabolic antenna having a circular mouth is to have a power gain of 1000 at λ = 10 cm. Estimate the diameter of the mouth and half power beam width of the antenna

Ans:

Given

Power gain (G_p) = 1000

$$\lambda = 10\ cm = 0.1\ m$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G_p = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$1000 = 6.4\left(\frac{D^2}{0.1^2}\right)$$

$$D = \sqrt{\frac{1000 \times 0.1^2}{6.4}} = 1.25\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.1}{1.25} = 4.64\ degree$$

7. A parabolic dish provides a gain of 75 dB at a frequency 15 GHz. Calculate the capture area of the antenna, its 3 dB and null beam widths

Ans:

Given

$$Gain(G) = 75\ dB$$

$$75 = 10 \log G$$

$$\frac{75}{10} = \log G$$

$$G = 10^{7.5} = 31622776.6$$

$$Frequency(f) = 15\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{15 \times 10^9} = 0.02\ m$$

Assume driven element is dipole ($\eta_{ap} = 0.65$)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65 \left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65 \left(\frac{\pi D^2}{4}\right) = 6.4 \left(\frac{D}{\lambda}\right)^2$$

$$31622776.6 = 6.4 \left(\frac{D}{\lambda}\right)^2 = 6.4 \frac{D^2}{\lambda^2}$$

$$D = \sqrt{\frac{31622776.6 \times 0.02^2}{6.4}} = 44.45\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.02}{44.45} = 0.02\ degree$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.02}{44.45} = 0.062\ degree$$

8. **A 64 meter diameter paraboloid reflector is operated at 1430 MHz and is fed by non directional antenna. Estimate beam width between half power points(HPBW) and between nulls(BWFN) and power gain w.r.t half wave dipole.**

Ans:

Given

Diameter (D) = 64 m

$$Frequency(f) = 1430\ MHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1430 \times 10^6} = 0.21\ m$$

$$HPBW = \frac{58\lambda}{D} = \frac{58 \times 0.21}{64} = 0.19\ degree$$

$$BWFN = \frac{140\ \lambda}{D} = \frac{140 \times 0.21}{64} = 0.46\ degree$$

For half wave dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65\left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{64}{0.21}\right)^2 = 594430.8$$

$$G_p = 10 \log (594430.8) = 57.7\ dB$$

9. **A paraboloid reflector antenna with diameter 20 m is designed to operate at a frequency of 6 GHz and illumination efficiency of 0.54. Calculate antenna gain in decibles.**

Ans:

Given

Diameter (D) = 20 m

$$Frequency(f) = 6\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05\ m$$

Illumination efficiency (η_{ap}) = 0.54

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.54 \left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.54 \left(\frac{\pi D^2}{4}\right) = 5.33\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 5.33\left(\frac{20}{0.05}\right)^2 = 852800$$

$$G_p = 10 \log (852800) = 59.3\ dB$$

10. Calculate beam width between first nulls of a 2.5 m paraboloid reflector used at 6 GHz. What will be its gain in decibels?

Ans:

Given

Diameter (D) = 2.5 m

$$Frequency(f) = 6\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05\ m$$

Assume driven element is dipole (η_{ap} = 0.65)

$$G_p = G = \frac{4\pi A_e}{\lambda^2}$$

$$But \qquad A_e = \eta_{ap} A_p = 0.65 \left(\frac{\pi D^2}{4}\right)$$

$$G = \frac{4\pi}{\lambda^2} \times 0.65 \left(\frac{\pi D^2}{4}\right) = 6.4\left(\frac{D}{\lambda}\right)^2$$

$$G_p = 6.4\left(\frac{2.5}{0.05}\right)^2 = 16000$$

$$G_p = 10 \log (16000) = 42\ dB$$

11. Calculate the angular aperture for paraboloid reflector antenna for which the aperture number is (i) 0.25 (ii) 0.5 (iii) 0.6 Given that diameter of the reflector mouth is 10 m, calculate the position of the focal point with reference to the reflector mouth in each case.

Ans:

(i)

$$\frac{f}{D} = 0.25$$
$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.25 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$\theta = 90^0$$

$$Angular\ aperture = 2\theta = 2 \times 90^0 = 180^0$$

$$Given\ (D) = 10\ m$$

$$\frac{f}{D} = 0.25$$
$$f = 0.25 \times D = 0.25 \times 10 = 2.5\ m$$

(ii)

$$\frac{f}{D} = 0.5$$
$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.5 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$\theta = 26.54^0$$

$$Angular\ aperture = 2\theta = 2 \times 26.54^0 = 53.08^0$$

$$Given\ (D) = 10\ m$$

$$\frac{f}{D} = 0.5$$
$$f = 0.5 \times D = 0.5 \times 10 = 5\ m$$

(iii)

$$\frac{f}{D} = 0.6$$

$$\frac{f}{D} = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$0.6 = \frac{1}{4}\cot\left(\frac{\theta}{2}\right)$$

$$\theta = 45.22^0$$

$$Angular\ aperture = 2\theta = 2 \times 45.22^0 = 90.44^0$$

$$Given\ (D) = 10\ m$$

$$\frac{f}{D} = 0.6$$

$$f = 0.6 \times D = 0.6 \times 10 = 6\ m$$

12. Estimate the diameter and the effective aperture of a paraboloid reflector antenna required to produce a nulls width of 10^0 at 3 GHz.

Ans:

Given

$$BWFN = 10^0$$

$$Frequency(f) = 3\ GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1\ m$$

$$BWFN = \frac{140\ \lambda}{D}\ degree$$

$$10 = \frac{140\ \times 0.1}{D}$$

$$D = \frac{140\ \times 0.1}{10} = 1.4\ m$$

ANTENNAS AND WAVE PROPAGATION

UNIT-IV (MICROSTRIP ANTENNAS AND ANTENNA MEASUREMENTS)

MICROSTRIP ANTENNAS

Features:

(i) The microstrip antenna was first proposed by G.A.Deschamps in 1953.
(ii) Microstrip antennas are similar to patch antennas.
(iii) The microstrip antenna is contained conducting strip or patch suspended over a ground plane
(iv) Microstrip antennas are simple to fabricate
(v) These antennas are constructed using lithographic patterning on a printed circuit boards.
(vi) The simplest patch antenna uses a half wavelength long patch with a larger ground plane.
(vii) The simple microstrip antenna generates a linearly polarized wave.
(viii) It is a narrowband, wide-beam antenna.
(ix) To achieve higher bandwidth, a relatively thick substrate is used.
(x) The microstrip antennas are often used where thickness and conformability to the surface of mount or platform are the key requirements.
(xi) The microstrip antennas are available with different shapes such as square, rectangular, circular, triangular or elliptical.
(xii) The size of the microstrip antenna is inversely proportional to its frequency.
(xiii) The following figure1 illustrate different shapes of microstrip antennas.

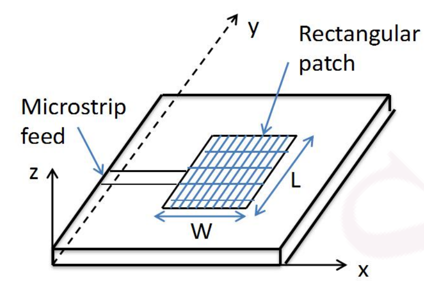


Fig.1: Rectangular patch antenna

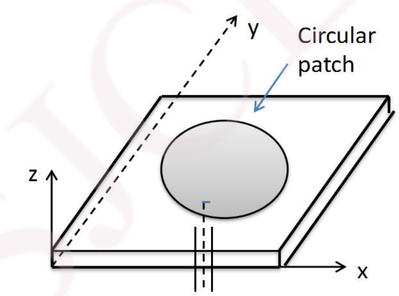


Fig.2: Circular patch antenna

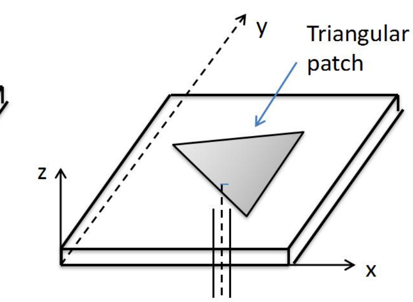


Fig.1: Triangular patch antenna

Advantages and limitations:

Advantages :

(i) Light weight
(ii) Smaller size
(iii) Lesser volume
(iv) Low profile planar configuration
(v) They can be easily molded to any desired shape
(vi) Simple to fabricate
(vii) Their fabrication process is compatible with Microwave Monolithic Integrated Circuit (MMIC) and Optoelectronic Integrated Circuit (OEIC) technologies.
(viii) These can support both linear and circular polarizations.
(ix) They are mechanically robust when mounted on rigid surfaces.
(x) With microstrip antennas it is easy to form large arrays with half wavelength or lesser spacing.

Limitations:

(i) Low bandwidth
(ii) Low efficiency
(iii) Low gain
(iv) Low power handling capacity

(v) Complicated design due to smaller size
(vi) These are resonant devices by its inherent nature
(vii) They suffer from the radiation effects due to feeds and junctions
(viii) These are poor end-fire radiators

BASIC CHARACTERISTICS OF MICROSTRIP ANTENNA:

1. *Radiation pattern:* The following figure shows radiation pattern of microstrip antenna in φ=0 direction and φ=90^0. The power radiated at 180^0 is about 15dB less than the power in the center of the beam i.e.at 90^0. The beam width is about 65^0 and the gain is about 9 dBi.

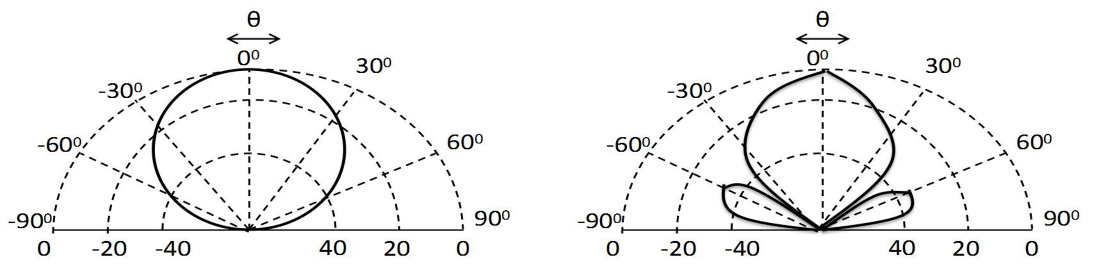


Fig 4(a): ϕ = 0⁰   Fig 4(b): ϕ = 90⁰

Fig 4: Radiation pattern for microstrip antenna

2. *Beam width:* The microstrip antennas have very wide bandwidth, both in azimuth and elevation.
3. *Directivity:* The directivity of microstrip antennas is given by
$$D = \frac{2h^2 E_0^2 W'^2 K_0^2}{P_r \pi \eta_0}$$
Where h thickness of the substrate, P_r is the radiated power, $W' = W + h$, $\eta_0 = 120\pi$, K_0 is the wave number and E_0 is the magnitude of the z-directed electric field.
4. *Gain:* The gain of the rectangular microstrip patch antenna will be in between 7 to 9 dB.
5. *Bandwidth:* The bandwidth decreases with increase of Quality factor. The impedance bandwidth of a patch antenna is influenced by the spacing between the patch and the ground palne. The bandwidth of the microstrip antenna is given by
$$Bandwidth = \frac{S-1}{Q_0\sqrt{S}}$$
Where S is the voltage standing wave ratio and Q_0 is the unloaded radiation quality factor.
Quality factor: The microstrip antennas have a very high quality factor. The quality factor 'Q' represents the losses associated with the antenna. When the quality factor is large then the bandwidth is low.
6. *Efficiency:* The loss factor of a microstrip antenna is given by
$$L_T = L_c + L_d + L_r$$
Where L_c is the conductor losses, L_d is the dielectric losses and L_r is the radiation losses. The efficiency of the microstrip antenna is given by
$$\eta = \frac{P_r}{P_c + P_d + P_r}$$
Where P_r is the radiated power, P_c is the power dissipated due to conductor losses, and P_d is the power dissipated due to dielectric.
7. *Polarization:* The main advantage of microstrip antenna is polarization diversity. The microstrip antennas can be designed to generate EM waves with different types of polarizations such as vertical, horizontal, circular polarizations. Circular polarization can be obtained from the patch antenna by exciting the square patch with two feeds with their inputs having 90^0 phase shift.

8. *Return loss:* The return loss is defined as the ratio of the Fourier transforms of the incident pulse and the reflected signal.
9. *Radar Cross-section:* The GPS guidance system requires low radar cross section (RCS) platforms. A standard technique used to reduce the RCS of a conventional patch antenna is to cover the patch with a magnetic absorbing material.

FEEDING METHODS:

There are many configurations that can be used to feed microstrip antennas. The four most popular are the microstrip line, coaxial probe, aperture coupling, and proximity coupling. These are displayed in Figure 14.3.

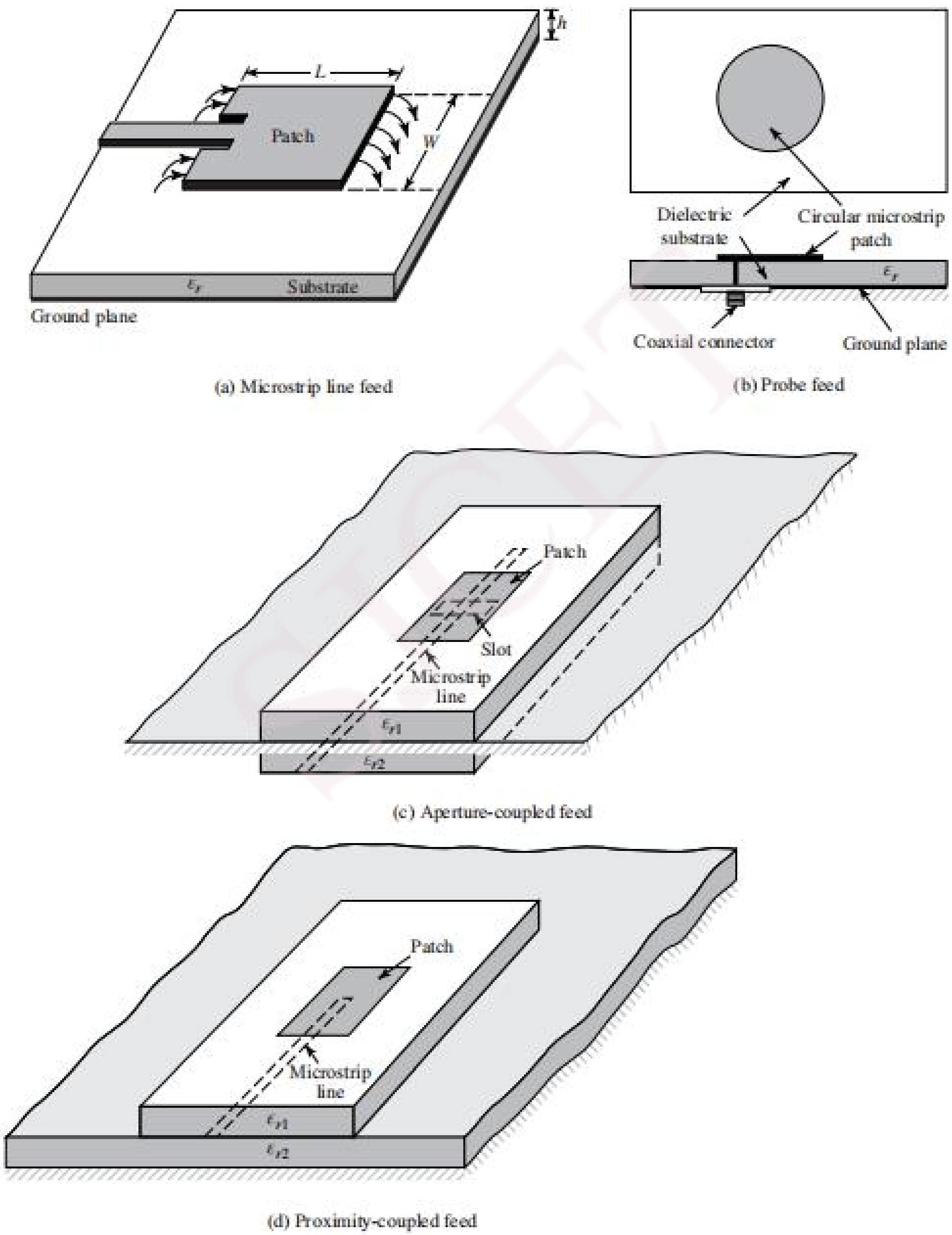


Figure 14.3 Typical feeds for microstrip antennas.

The microstrip feed line is also a conducting strip, usually of much smaller width compared to the patch. The microstrip-line feed is easy to fabricate, simple to match by controlling the inset position and rather simple to model. However as the substrate thickness increases, surface waves and spurious feed radiation increase, which for practical designs limit the bandwidth.

Coaxial-line feeds, where the inner conductor of the coax is attached to the radiation patch while the outer conductor is connected to the ground plane, are also widely used. The coaxial probe feed is also easy to fabricate and match, and it has low spurious radiation. However, it also has narrow bandwidth and it is more difficult to model.

Both the microstrip feed line and the probe possess inherent asymmetries which generate higher order modes which produce cross-polarized radiation. To overcome some of these problems,noncontacting aperture-coupling feeds, as shown in Figures 14.3(c,d), have been introduced. The aperture coupling of Figure 14.3(c) is the most difficult of all four to fabricate and it also has narrow bandwidth. However, it is somewhat easier to model and has moderate spurious radiation. The aperture coupling consists of two substrates separated by a ground plane. On the bottom side of the lower substrate there is a microstrip feed line whose energy is coupled to the patch through a slot on the ground plane separating the two substrates. This arrangement allows independent optimization of the feed mechanism and the radiating element. Typically a high dielectric material is used for the bottom substrate, and thick low dielectric constant material for the top substrate. The ground plane between the substrates also isolates the feed from the radiating element and minimizes interference of spurious radiation for pattern formation and polarization purity. For this design, the substrate electrical parameters, feed line width, and slot size and position can be used to optimize the design.Typically matching is performed by controlling the width of the feed line and the length of the slot.

METHODS OF ANALYSIS

There are many methods of analysis for microstrip antennas. The most popular models are the *transmission-line*, *cavity* , and *full wave* models.The transmission-line model is the easiest of all, it gives good physical insight, but is less accurate and it is more difficult to model coupling. Compared to the transmission-line model, the cavity model is more accurate but at the same time more complex. However, it also gives good physical insight and is rather difficult to model coupling, although it has been used successfully. In general when applied properly, the full-wave models are very accurate, very versatile, and can treat single elements, finite and infinite arrays, stacked elements, arbitrary shaped elements, and coupling. However they are the most complex models and usually give less physical insight.

Design of Rectangular patch antennas:

The rectangular patch is by far the most widely used configuration. It is very easy to analyze using both the transmission-line and cavity models, which are most accurate for thin substrates.

A design procedure is outlined which leads to practical designs of rectangular microstrip antennas. The procedure assumes that the specified information includes the dielectric constant of the substrate (*r*), the resonant frequency (*fr*), and the height of the substrate *h*. The procedure is as follows:

Specify:

$$\varepsilon_r, f_r(in\ Hz), and\ h$$

Determine:

$$\mathrm{W, L}$$

Design Procedure:

(1) For an efficient radiator, a practical width that leads to good radiation efficiencies is

$$W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{v_0}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}}$$

where v_0 is the free-space velocity of light.

(2) Determine the effective dielectric constant of the microstrip antenna using

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}$$

(3) Once W is found using, determine the extension of the length ΔL using

$$\Delta L = 0.412(h)\frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$$

(4) The actual length of the patch can now be determined by using

$$L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} - 2\Delta L$$

Typical lengths of microstrip patches vary between

$$L \approx (0.47 - 0.49)\frac{\lambda_0}{\sqrt{\varepsilon_r}} = (0.47 - 0.49)\lambda_d$$

where λd is the wavelength in the dielectric. The smaller the dielectric constant of the substrate, the larger is the fringing; thus, the length of the microstrip patch is smaller. In contrast, the larger the dielectric constant, the more tightly the fields are held within the substrate; thus, the fringing is smaller and the length is longer and closer to half-wavelength in the dielectric.

Design of Circular Patch Antennas

Other than the rectangular patch, the next most popular configuration is the circular patch or disk, as shown in Figure 14.24.

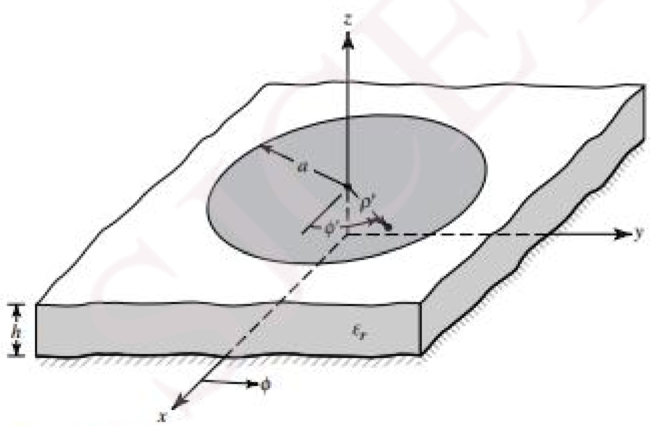


Figure 14.24 Geometry of circular microstrip patch antenna.

The modes supported by the circular patch antenna can be found by treating the patch, ground plane, and the material between the two as a circular cavity.

Based on the cavity model formulation, a design procedure is outlined which leads to practical designs of circular microstrip antennas for the dominant TM^z_{110} mode. The procedure assumes that the specified information includes the dielectric constant of the substrate (*r*), the resonant frequency (*fr*) and the height of the substrate *h*. The procedure is as follows:

Specify:

$$\varepsilon_r, f_r(in\ Hz), and\ h\ (in\ cm)$$

Determine The actual radius *a* of the patch.

Design Procedure:

(1) Determine the radius of circular patch by using

$$a = \frac{F}{\left\{1 + \frac{2h}{\pi\varepsilon_r F}\left[\ln\left(\frac{\pi F}{2h}\right) + 1.7726\right]\right\}^{1/2}}$$

Where

$$F = \frac{8.791 \times 10^9}{f_r\sqrt{\varepsilon_r}}$$

Remember that h in above equations must be in cm.

<u>INTRODUCTION TO SMART ANTENNAS-CONCEPT OF ADAPTIVE BEAM FORMING</u>

The functionality of many engineering systems is readily understood when it is related to our human body system. Therefore, to give an insight into how a smart-antenna system works, let us imagine two persons carrying on a conversation inside a dark room [refer to Figure 16.1(a)].

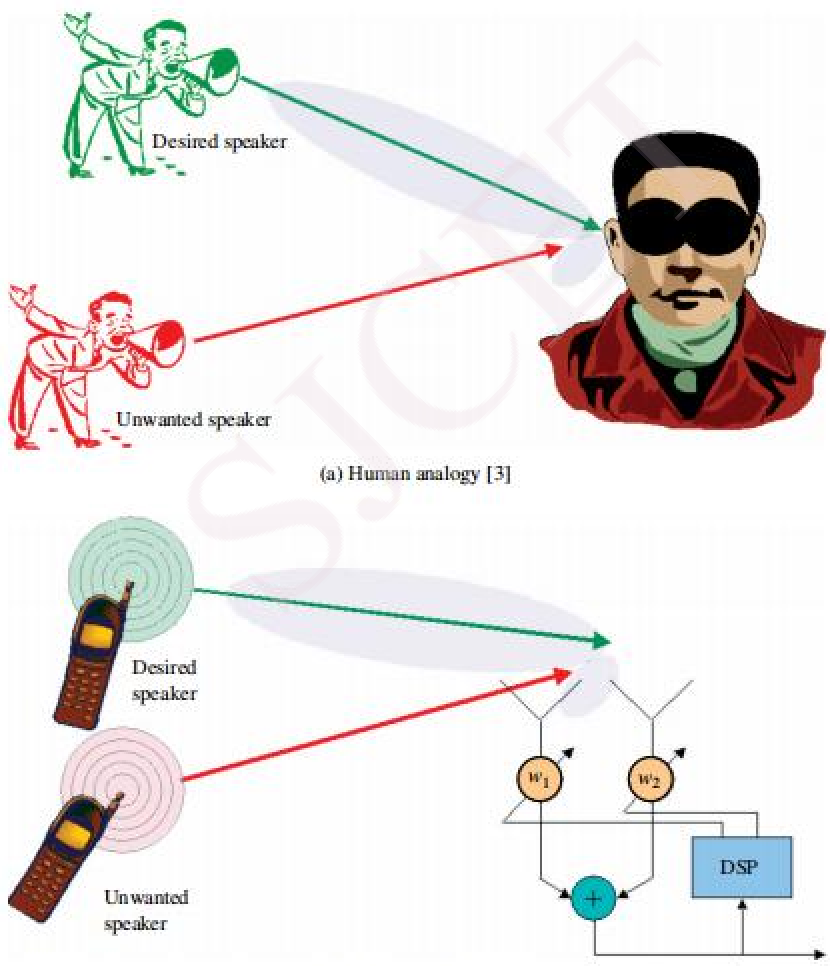


Figure 16.1 Smart-antenna analogy. (a) Human analogy [3]; (b) Electrical equivalent.

The listener among the two persons is capable of determining the location of the speaker as he moves about the room because the voice of the speaker arrives at each acoustic sensor, the ear, at a different time. The human signal processor, the brain, computes the direction of the speaker from the time differences or delays of the voice received by the two ears. Afterward, the brain adds the strength of the signals from each ear so as to focus on the sound of the computed direction. Furthermore, if additional speakers join in the conversation, the brain can tune out unwanted interferers and concentrate on one conversation at a time. Conversely, the listener can respond back to the same direction of the desired speaker by orienting the transmitter (mouth) toward the speaker.

Electrical smart-antenna systems work the same way using two antennas instead of the two ears and a digital signal processor instead of a brain [refer to Figure 16.1(b)]. Therefore, after the digital signal processor measures the time delays from each antenna element, it computes the direction of arrival (DOA) of the signal-of-interest (SOI), and then it adjusts the excitations (gains and phases of the signals) to produce a radiation pattern that focuses on the SOI while, ideally, tuning out any signal-not-of-interest (SNOI).

Maintaining capacity has always been a challenge as the number of services and subscribers increased. To achieve the capacity demand required by the growing number of subscribers, cellular radio systems had to evolve throughout the years. To justify the need for smart-antenna systems in the current cellular system structure, a brief history on the evolution of the cellular radio systems is presented.

Adaptive Beam Forming:

As discussed in earlier, the information supplied by the DOA algorithm is processed by means of an adaptive algorithm to ideally steer the maximum radiation of the antenna pattern toward the SOI and place nulls in the pattern toward the SNOIs. *This is only necessary for DOA-based adaptive beamforming algorithms. However, for reference (or training) based adaptive beamforming algorithms, like the Least Mean Square (LMS) that is used in this chapter, the adaptive beamforming algorithm does not need the DOA information but instead uses the reference signal, or training sequence, to adjust the magnitudes and phases of each weight to match the time delays created by the impinging signals into the array.* In essence, this requires solving a linear system of normal equations. The main reason why it is generally undesirable to solve the normal equations directly is because the signal environment is constantly changing. Before reviewing the most common adaptive algorithm used in smart antennas, an example is given, based on, to illustrate the basic concept of how the weights are computed to satisfy certain requirements of the pattern, especially the formation of nulls.

MEASUREMENT OF ANTENNA PARAMETERS

BASIC SETUP AND RADIATION PATTERN MEASUREMENT

The arrangement for measurement of radiation pattern is shown in the figure below. It contains transmitting antenna primary antenna, Antenna Under Test (AUT) called secondary antenna, mount for rotating the primary antenna, detector or receiver and indicator. The primary antenna will radiate the signal towards the secondary antenna. The secondary antenna will be rotated with the help of antenna drive unit. The indicator will be used to indicate or to measure the relative magnitude of the received field. There are two requirements for conducting the experiments with the above arrangement such as distance requirement and uniform illumination requirement.

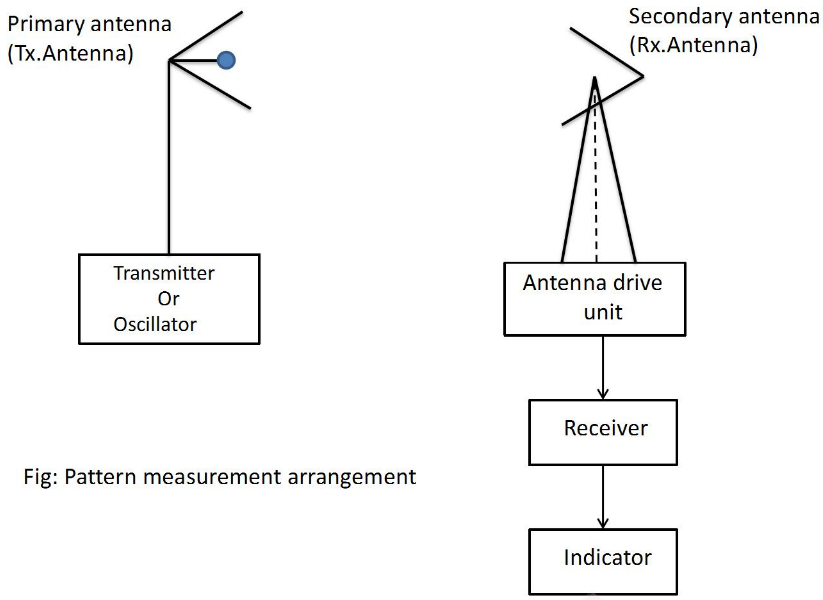


Fig: Pattern measurement arrangement

The distance between the two antennas must be related to the following equation

$$r \geq \frac{2d^2}{\lambda}$$

Or

$$r = \frac{d^2}{8\delta}$$

Where d is the maximum linear dimension of the either antenna, λ is the wavelength and δ is phase difference error.

The other requirement for an accurate field pattern measurement is, the primary antennas should produce a plane of wave of uniform amplitude and phase over the distance r.

<u>DIRECTIVITY MEASUREMENT</u>

The directivity of antenna is defined as

$$Directivity(D) = \frac{Maximum\ radiation\ intensity}{Average\ radiaiton\ intensity}$$

$$D = \frac{4\pi \times Maximum\ radiaiton\ intensity}{\int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} Radiaiton\ intensity \times \sin\theta d\varphi}$$

Form the above equation the directivity can be determined knowing the radiation intensity with the help of radiation pattern. In the above equation the integral part in the denominator can be determined using any one of the following two methods:

Orange slice method:

In this method, set of patterns is obtained by measuring the radiation intensity versus θ for a discrete value of φ. Then each pattern is multiplied continuously by sinθ weight factor and then summed together.

Conical cut method:

In this method, set of patterns is obtained by measuring the radiation intensity versus φ for a discrete value of θ. Then each pattern is multiplied continuously by sinθ weight factor and then summed together.

<u>GAIN MEASUREMENT</u>

<u>Gain measurement by using comparison method</u>:

The set up require for the measurement of gain of the antenna by using the comparison method is shown in the figure below

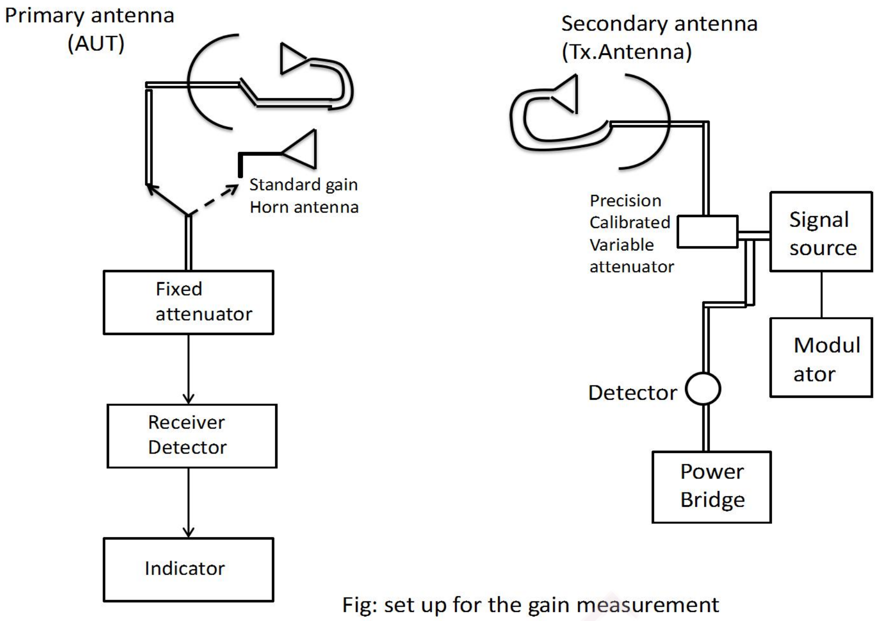


Fig: set up for the gain measurement

The transmitting section contains, signal source, modulator, precision calibrated variable attenuator, detector, Power Bridge and arbitrary transmitting antenna. The modulator modulates the signal generated by the signal source. This modulated signal is transmitted towards the receiving antenna. The precision calibrated variable attenuator is used to adjust the power to the required level. The power bridge is used to sense the variations in the frequency of operation if any during the experiment. The receiver section contains two antennas such as antenna under test (AUT) and standard gain horn antenna (reference antenna), fixed attenuator, receiver detector and indicator. The purpose of fixed attenuator is, to avoid the impedance mismatch between the two antennas and the receiver. The experimental procedure is explained as follows:

(i) Connect the standard horn antenna to the receiver with the help of switch‘s’ and orient the antenna towards the transmitting antenna.

(ii) Adjust the input to the transmitting antenna to a convenient level with the help of precision calibrated variable attenuator.

(iii) Note down the attenuator dial setting and let it be W_1.

(iv) Note down the reading of Power Bridge and let it be P_1.

(v) Now replace the standard horn antenna with AUT (antenna under test).

(vi) Again adjust the precision calibrated variable attenuator such that, the receiver indicates the same previous reading as was with horn antenna.

(vii) Again note down the readings of variable attenuator and power Bridge and let it be W_2 and P_2 respectively.

(viii) When $P_1 = P_2$, then calculate the gain by using the formula

$$Gain(G) = \frac{W_2}{W_1}$$

Or gain in dB is $G(dB) = W_2(dB) - W_1(dB)$

(ix) When $P_1 \neq P_2$, then calculate the gain by using the formula

$$G = \frac{W_2}{W_1} \times \frac{P_1}{P_2} = G_p \times P$$

Or gain in dB is $G(dB) = G_p(dB) + P(dB)$

<u>Gain Measurement by using Absolute method:</u>

The experimental set up for the measurement of gain by using the absolute method is shown the figure below.

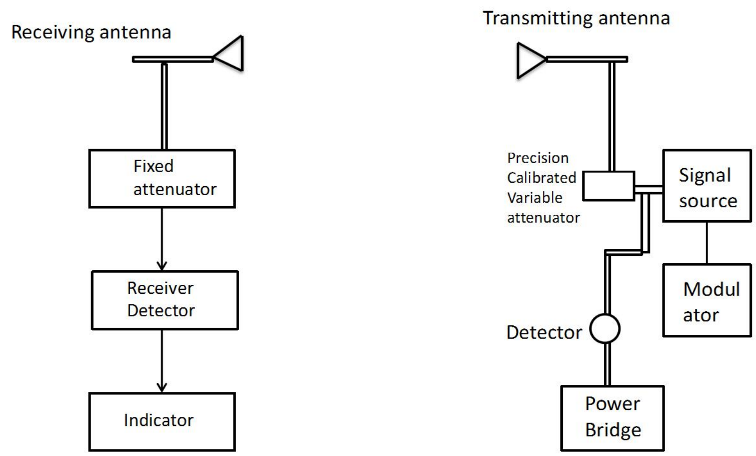


Fig: set up for the gain measurement

The transmitting section contains, signal source, modulator, precision calibrated variable attenuator, detector, Power Bridge and transmitting antenna. The modulator modulates the signal generated by the signal source. This modulated signal is transmitted towards the receiving antenna. The precision calibrated variable attenuator is used to adjust the power to the required level. The power bridge is used to sense the variations in the frequency of operation if any during the experiment. The receiver section contains receiving antenna, attenuator pad or fixed attenuator, receiver detector and indicator. In this method, the two antennas (transmitting and receiving) must be identical. The experimental procedure is explained as follows:

(i) Transmit the power with transmitting antenna towards the receiving antenna with the help of signal source and let this transmitted power is P_T.

(ii) Note down the power received with the receiving antenna with the help of receiver and indicator and let it be P_R.

(iii) Calculate the value of gain by using the Friiss transmission equation given by

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

Where P_R = Power received

P_T = Power transmitted

G_T = Gain of the transmitting antenna

G_R = Gain of the receiving antenna

R = Distance between the two antennas

λ = Wavelength

Since the two antennas (transmitting and receiving) are identical, $G_T = G_R = G$

Therefore the gain (G) is given by

$$G = \frac{4\pi R}{\lambda}\sqrt{\frac{P_R}{P_T}}$$

Gain Measurement by using 3-Antenna Method:

When two identical antennas are not available, then three antenna method will be used to measure the gain of the antenna. In three antenna method, the gain will be measured by using three different antennas. The experimental set up for the measurement of gain by using three antenna method is shown in the figure below. In this method, three equations will be obtained with the help of three antennas. Let the three antennas are A_1, A_2 and A_3. Three equations will be obtained as follows:

Case-I (Antenna A_1 is transmitting and antenna A_2 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R2} = P_{T1}G_{T1}G_{R2}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 1$$

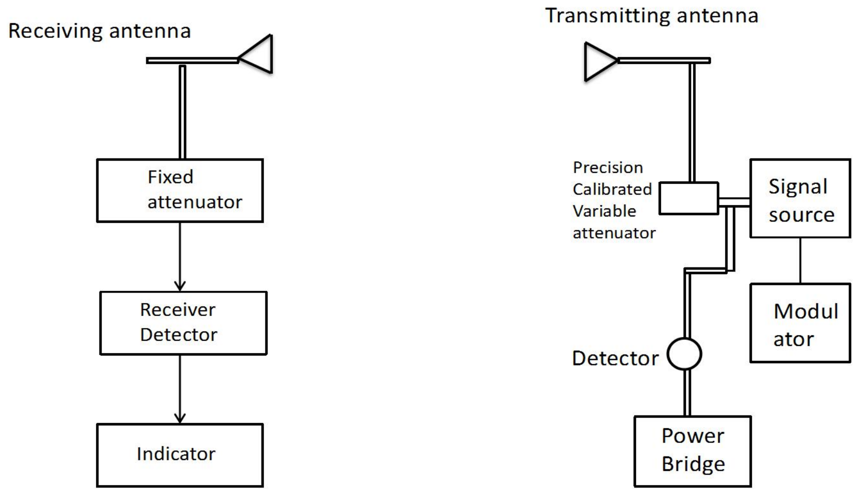


Fig: set up for the gain measurement

Where P_{T1} is the power transmitted by antenna A_1, P_{R2} is the power received by the antenna A_2, G_{T1} is the gain of the antenna A_1, G_{R2} is the gain of the receiving antenna A_2 and R is the distance between the transmitting and receiving antenna.

Case-II (Antenna A_2 is transmitting and antenna A_3 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R3} = P_{T2}G_{T2}G_{R3}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 2$$

Where P_{T2} is the power transmitted by antenna A_2, P_{R3} is the power received by the antenna A_3, G_{T2} is the gain of the antenna A_2, G_{R3} is the gain of the receiving antenna A_3 and R is the distance between the transmitting and receiving antenna.

Case-III (Antenna A_3 is transmitting and antenna A_1 is receiving):

By using the friiss transmission equation the, the relation between the received power and transmitted power is given by

$$P_{R1} = P_{T3}G_{T3}G_{R1}\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 3$$

Where P_{T3} is the power transmitted by antenna A_3, P_{R1} is the power received by the antenna A_1, G_{T3} is the gain of the antenna A_3, G_{R1} is the gain of the receiving antenna A_1 and R is the distance between the transmitting and receiving antenna.

But

$$G_{T1} = G_{R1} = G_1$$
$$G_{T2} = G_{R2} = G_2$$
$$G_{T3} = G_{R3} = G_3$$

Then the above three equations (1, 2, 3) becomes

$$P_{R2} = P_{T1}G_1G_2\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 4$$
$$P_{R3} = P_{T2}G_2G_3\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 5$$
$$P_{R1} = P_{T3}G_3G_1\left(\frac{\lambda}{4\pi R}\right)^2 \qquad -- 6$$

Equations 4, 5 and 6 consist of three unknowns such as G_1, G_2, and G_3. By solving these three equations we can obtained the gain of any antenna (either the gain of A_1 or A_2 or A_3).

SOLVED PROBLEMS

1. Design a rectangular microstrip antenna using a substrate (RT/duroid 5880) with dielectric constant of 2.2, *h* = 0.1588 cm (0.0625 inches) so as to resonate at 10 GHz.

***Solution*:**

The width W of the patch is

$$W = \frac{v_0}{2f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

$$W = \frac{3 \times 10^8}{2 \times 10 \times 10^9}\sqrt{\frac{2}{2.2 + 1}} = 1.186 \text{ cm}$$

The effective dielectric constant of the patch is

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}$$

$$\varepsilon_{reff} = \frac{2.2 + 1}{2} + \frac{2.2 - 1}{2}\left[1 + 12\frac{0.1588}{1.186}\right]^{-1/2} = 1.972$$

The extended incremental length of the patch ΔL is

$$\Delta L = 0.412(h)\frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$$

$$\Delta L = 0.412(0.1588)\frac{(1.972 + 0.3)\left(\frac{1.186}{0.1588} + 0.264\right)}{(1.972 - 0.258)\left(\frac{1.186}{0.1588} + 0.8\right)} = 0.081 \text{ cm}$$

The actual length L of the patch is

$$L = \frac{\lambda}{2} - 2\Delta L = \frac{v_0}{2f_r\sqrt{\varepsilon_{reff}}} - 2\Delta L = \frac{3 \times 10^8}{2 \times 10 \times 10^9\sqrt{1.972}} - 2 \times 0.081 = 0.906 \text{ cm}$$

Finally the effective length is

$$L_e = L + 2\Delta L = 0.906 + 2(0.081) = 1.068\ cm$$

2. Design a circular microstrip antenna using a substrate (RT/duroid 5880) with a dielectric constant of 2.2, *h* = 0.1588 cm (0.0625 in.) so as to resonate at 10 GHz.

***Solution*:**

$$F = \frac{8.791 \times 10^9}{f_r\sqrt{\varepsilon_r}} = \frac{8.791 \times 10^9}{10 \times 10^9\sqrt{2.2}} = 0.593$$

$$a = \frac{F}{\left\{1 + \frac{2h}{\pi\varepsilon_r F}\left[\ln\left(\frac{\pi F}{2h}\right) + 1.7726\right]\right\}^{1/2}}$$

$$a = \frac{0.593}{\left\{1 + \frac{2 \times 0.1588}{\pi \times 2.2 \times 0.593}\left[\ln\left(\frac{\pi \times 0.593}{2 \times 0.1588}\right) + 1.7726\right]\right\}^{1/2}} = 0.525 \text{ cm}$$

ANTENNAS AND WAVE PROPAGATION

UNIT-V (WAVE PROPAGATION)

DEFINITION OF WAVE

Generally when a wave travelling in any dielectric medium, it will be described in terms of equation known as wave equation or Helmholtz equation. The wave equation for the dielectric medium is given by

$$\nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 H = \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

In general a wave is defined as follows:
"If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location, the group of phenomena constitutes a wave".

CHARACTERIZATIONS AND GENERAL CLASSIFICATION OF WAVES

Characterizations: In general the waves have been categorized in to two types such as guided waves and unguided waves.

Guided waves: When the waves are guided by any guiding devices like cables, waveguides, transmission lines, optical fibers, etc., then they are known as guided waves. Guided waves can be found in the applications like LAN (Local Area Networks) cables, Closed circuit TV, Cable network used by the cable TV operators, etc.

Unguided waves: When the waves are not guided by any guiding devices, then they are known as unguided waves. Unguided waves can be found in the applications like Radar systems, Satellite communications, TV, Terrestrial communications, Radio broadcasting, etc. The free space signal is nothing but a unguided wave.

Classification of Electromagnetic waves:

(a) General classification:

(1) Plane wave: A wave is said to be plane wave if it maintains constant phase at each and every point.

(2)Uniform plane wave: A wave is said to be uniform plane wave if it maintains constant phase and amplitude at each and every point.

(3)Non-uniform plane wave: A wave is said to be non-uniform plane wave if it do not maintains constant phase and amplitude at each and every point.

(4) Slow wave: When a wave travels with velocity less than the free space velocity©, then it is known as slow wave.

(5)Forward wave: A wave traveling in a assigned direction from the point of origin, then it is known as forward wave.

(6)Backward wave: The backward wave is, in general, a reflected wave which results, when a forward wave strikes a reflecting surface.

(7)Traveling wave: When a wave propagating in only one direction and there is no reflected wave present, then it is called as a traveling wave.

(8)Standing wave: If both forward wave and reflected wave are simultaneously present, then they combine to results in a wave called standing wave. In standing wave, the wave does not progress but its amplitude will increase with distance and time.

(9)Surface wave: When a wave travels along the surface of any media like earth surface, then it is known as surface wave.

(10)Trapped wave: Sometimes a surface wave is also called as a trapped wave because it carries its energy with in a small distance from the interface. When a wave travels between two reflecting layers due to trapping, then it is known as trapped wave.

(11)Leaky wave: when discontinuities are densely placed along the line, another type of traveling wave results and are called as leaky wave.

(b) <u>Classification based on orientation of field vector:</u>

(1)Horizontally polarized wave: When the electric filed strength vector (E) is oriented in horizontal direction then it is known as horizontally polarized wave.

(2)Vertically polarized wave: When the electric filed strength vector (E) is oriented in vertical direction then it is known as vertically polarized wave.

(3)Elliptically polarized wave: When a wave contains the elliptical polarization, then it is known as elliptically polarized wave. For elliptical polarization, the minimum condition is two components of electric field must have unequal amplitudes any ohase difference other than zero.

(4)Circularly polarized wave: When a wave contains the circular polarization, then it is known as circularly polarized wave. For circular polarization, the minimum condition is two components of electric field must have equal amplitudes 90^0 phase difference.

(c) <u>Classification based on presence of filed components:</u>

(1)TE waves or H-Waves: When a wave does not contain the electric field component in the direction of propagation, then it is known as transverse electric wave or simply H-wave.

(2)TM waves or E-Waves: When a wave does not contain the magnetic field component in the direction of propagation, then it is known as transverse magnetic wave or simply E-wave.

(3)TEM waves: When a wave does not contain either the electric field component or the magnetic field component in the direction of propagation, then it is known as transverse electromagnetic wave.

(d) <u>Classification based on modes of propagation:</u>

(1)Ground wave or surface wave: When a wave propagates along the earth surface then it is known as ground wave or surface wave.

(2)Space wave or tropospheric wave: When a wave travels through the troposheric layer of the atmosphere, then it is known as space wave or tropospheric wave.

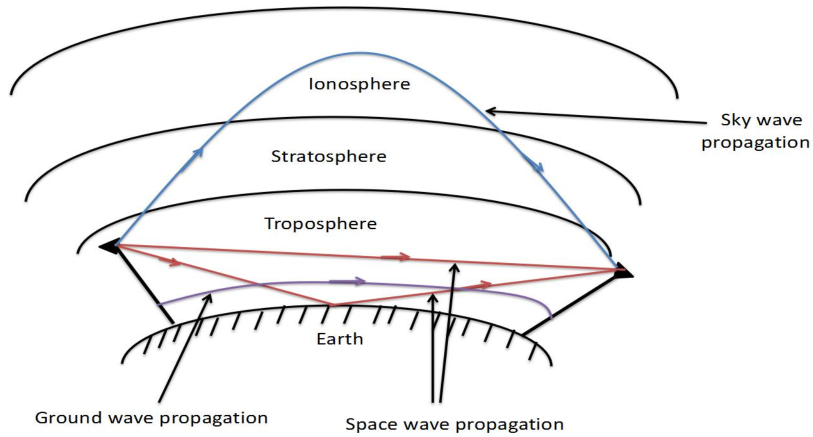
(3)Sky wave or Ionospheric wave: When a wave travels through the ionospheric layer of the atmosphere, then it is known as sky wave or ionospheric wave.

<u>DIFFERENT MODES OF WAVE PROPAGATION</u>

There are three modes of wave propagation such as

(i) Ground wave or Surface wave propagation
(ii) Space wave or Tropospheric wave propagation
(iii) Sky wave or Ionospheric wave propagation

These three modes are represented in the figure shown below



Ground wave or Surface wave propagation: When the waves propagate along the surface of the earth, then it is known as ground wave or surface wave propagation. The ground wave propagation will be used for the frequencies less than 2 MHz. The ground wave propagation covers more distance as compared with the space wave propagation. The ground wave field strength dies out after traveling long distance due to wavefront tilting and earth attenuation.

Space wave or Tropospheric wave propagation: When the waves propagate through the tropospheric layer of atmosphere then it is known as space wave or tropospheric wave propagation. This propagation will be used for the frequency greater than the 30 MHz. This propagation is also called as LOS (Line Of Sight) propagation. It covers less distance as compared with ground wave and sky wave propagation. Space waves do not follow the earth's curvature and hence they cannot travel more distance.

Sky wave or Ionospheric wave propagation: When the waves propagate through the Ionospheric layer of atmosphere then it is known as sky wave or ionospheric wave propagation. This propagation will be used for the frequency between 2 MHz to 30 MHz. It covers very long distance as compared with ground wave and sky wave propagation.

RAY/MODE CONCEPTS

Ray is defined as the perpendicular drawn to an equiphase plane and where as the mode is defined as the field distribution or configuration present in the wave. There are two concepts for analyzing any wave such as ray concept or ray theory and mode concept or mode theory. Ray theory is applicable when the distance between two reflecting layers is large where as mode theory will be used when the distance between the two reflecting layers is small. Ray theory will be used when the distance between the two antennas is small where as mode theory is used when the distance between the two antennas is large. When using ray theory, a surface wave needs to be separately taken in to account.

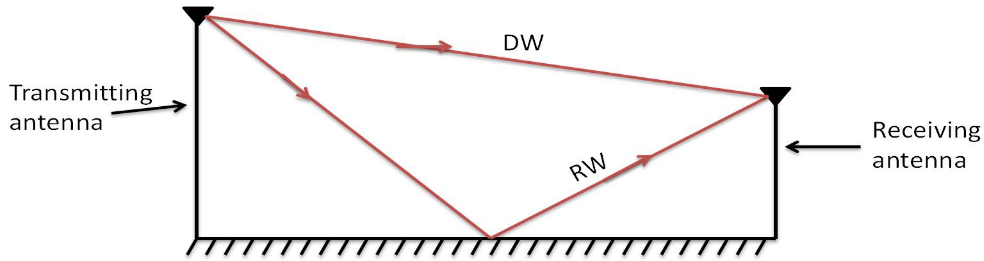
GROUND WAVE PROPAGATION

Introduction:

When the waves propagate along the surface of the earth, then it is known as ground wave or surface wave propagation. The ground wave propagation will be used for the frequencies less than 2 MHz. The ground wave propagation covers more distance as compared with the space wave propagation. The ground wave field strength dies out after traveling long distance due to wavefront tilting and earth attenuation.

Plane earth reflections:

When a wave is reflected from the flat earth or plane earth, then it is known as plane earth reflections. When the distance between the transmitting and receiving antenna is small, then the earth can be imagined as flat or plane surface. The plane earth reflections are shown in the figure below.



From the above figure it can be observed that, the signal strength at the receiving point is the combination of direct wave and reflected wave. The magnitude of the reflected signal depends upon the type of earth surface i.e smooth surface or rough surface. The roughness of the earth can be calculated based on the following empherical formula.

$$R = 4\pi\sigma sin\theta/\lambda$$

Where

σ is the standard deviation of the surface irregularities

θ is the angle of incidence with respect to the normal to the earth surface

λ is the wavelength.

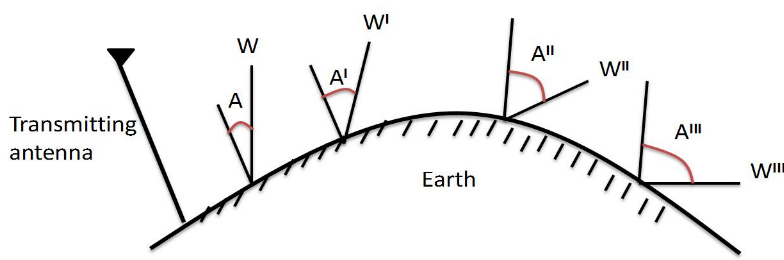
When the value of R is less than 0.1, then the earth surface can be considered as smooth and when it is greater than 10, then it is considered as rough surface. When the angle θ is zero, then the surface is smooth. When the signal is reflected from the smooth surface, then the amplitude of the reflected signal will be equal to the incident signal where as if it reflected from the rough surface, the amplitude of the reflected signal will be less than the incident signal because of scattering of signal by the rough surface.

Space and surface waves:

According to sommerfeld, the ground wave is divided in to two parts such as space wave and surface wave. The space wave exist at certain height from the earth surface where as surface wave exist on the surface of the earth itself. The space wave is combination of direct wave and reflected wave. The surface wave is also called as ground wave.

Wave tilt:

The following figure represents the basic principle involved in ground wave propagation.

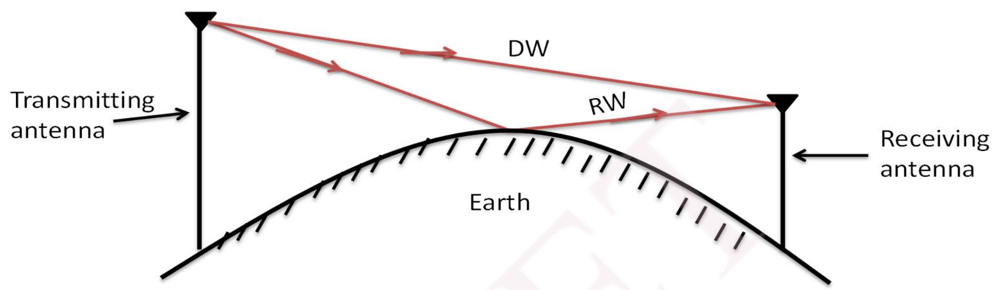


In ground wave propagation, the waves glides over the earth surface i.e the waves travels along the surface of the earth. In this propagation, the lower tip of the electric filed strength vector will be in touch with the earth surface. Initially if we assume the orientation of electric vector is vertical, later it becomes horizontal after traveling long distance due to the tilting of wavefront. Due to the surface irregularities of the earth, the wavefront of the waves will tilt as shown in the figure. Therefore, the ground wave field strength will be dies out after traveling certain distance due to shorting out the horizontal component by the earth(earth is assumed as good conductor).

Curved earth reflections:

When a wave is reflected from the curved earth or plane earth, then it is known as curved earth reflections. When the distance between the transmitting and receiving antenna is large, then the earth can be imagined as curved earth. The curved earth reflections are shown in the figure below.

Due to the curvature of the earth, the space wave signal will be affected more as compared with the surface wave signal. Form the above figure it can be observed that, the physical heights of the antennas will be more than the effective heights. The major difference between the plane earth reflections and curved earth reflections is, the path difference between the direct wave and reflected wave will be more in plane earth reflections as compared with curved earth reflections.



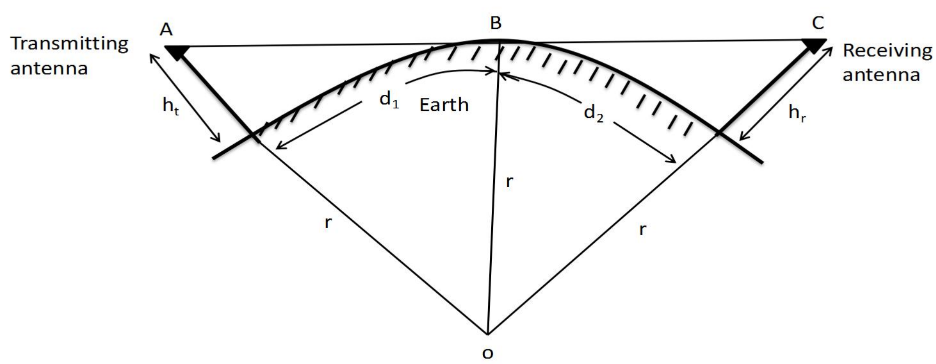
SPACE WAVE PROPAGATION

Introduction:

When the waves propagate through the tropospheric layer of atmosphere then it is known as space wave or tropospheric wave propagation. This propagation will be used for the frequency greater than the 30 MHz. This propagation is also called as LOS (Line Of Sight) propagation. It covers less distance as compared with ground wave and sky wave propagation. Space waves do not follow the earth's curvature and hence they cannot travel more distance.

LOS range or coverage distance:

The coverage range or LOS distance is defined as the maximum distance at which the space wave signal would be received with receiving antenna. The LOS range can be calculated from the figure shown below.



From triangle OAB,

$$(r + h_t)^2 = r^2 + d_1^2$$
$$d_1^2 = (r + h_t)^2 - r^2$$

$$d_1 = \sqrt{(r + h_t)^2 - r^2}$$
$$d_1 = \sqrt{r^2 + 2rh_t + h_t^2 - r^2}$$
$$d_1 = \sqrt{2rh_t} \qquad -1$$

h_t^2 is neglected as compared with $2Rh_t$

Similarly from triangle OBC,

$$(r + h_r)^2 = r^2 + d_2^2$$
$$d_2^2 = (r + h_r)^2 - r^2$$
$$d_2 = \sqrt{(r + h_r)^2 - r^2}$$
$$d_2 = \sqrt{r^2 + 2rh_r + h_r^2 - r^2}$$
$$d_1 = \sqrt{2rh_r} \qquad -2$$

h_r^2 is neglected as compared with $2Rh_r$

$$LOS\ range\ (d) = d_1 + d_2 \qquad -3$$

Substitute equations 1 and 2 in equation 3

Then,

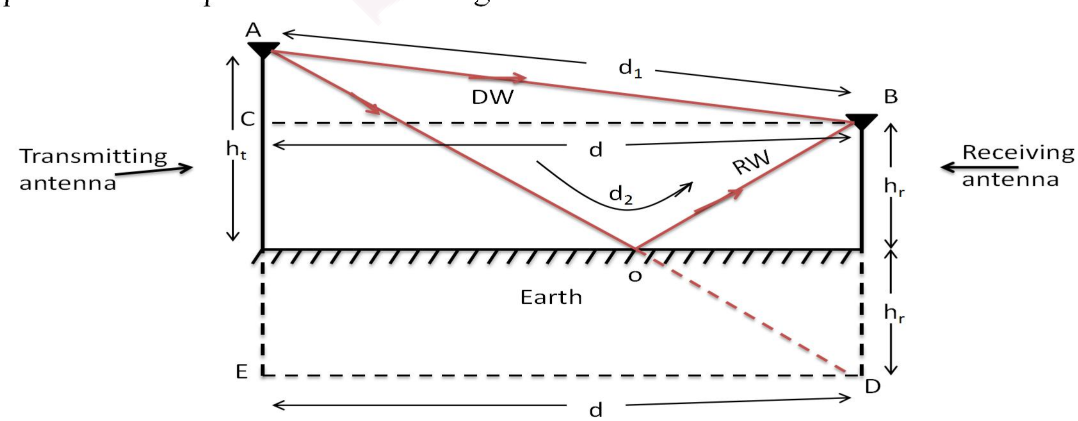
$$d = \sqrt{2rh_t} + \sqrt{2rh_r}$$
$$d = \sqrt{2r}\left(\sqrt{h_t} + \sqrt{h_r}\right) \quad in\ meters$$

In above equation the letter ‘r’ represents the earth radius which will be equal to 6370 km. The above equation can be expressed in kilometers as

$$d = \sqrt{2 \times 6370\ km}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6370 \times 10^3}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6.37 \times 10^3 \times 10^3}\left(\sqrt{h_t} + \sqrt{h_r}\right)$$
$$d = \sqrt{2 \times 6.37}\left(\sqrt{h_t} + \sqrt{h_r}\right) \times 10^3$$
$$d = 3.57\left(\sqrt{h_t} + \sqrt{h_r}\right) \quad km$$

Field strength variation with distance and height:

The equation for the space wave field strength is derived as follows:



The path difference between the direct wave and reflected wave is given by

$$P.d = d_2 - d_1 \qquad -1$$

Form the Δ OAB shown in above figure,

$$d_1^2 = d^2 + (h_t - h_r)^2$$

$$d_1^2 = d^2\left(1 + \left(\frac{h_t - h_r}{d}\right)^2\right)$$

$$d_1 = d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2} = d\left[1 + \left(\frac{h_t - h_r}{d}\right)^2\right]^{1/2}$$

Expand by using binomial expansion

$$d_2 = d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right]$$

$$d_1 = d + \frac{(h_t - h_r)^2}{2d} \qquad -2$$

Similarly from ΔOBC,

$$d_2^2 = d^2 + (h_t + h_r)^2$$

$$d_2^2 = d^2\left(1 + \left(\frac{h_t + h_r}{d}\right)^2\right)$$

$$d_2 = d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} = d\left[1 + \left(\frac{h_t + h_r}{d}\right)^2\right]^{1/2}$$

Expand by using binomial expansion

$$d_2 = d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right]$$

$$d_2 = d + \frac{(h_t + h_r)^2}{2d} \qquad -3$$

Substitute equations 2 and 3 in equation 1

$$p.d = d + \frac{(h_t + h_r)^2}{2d} - \left(d + \frac{(h_t - h_r)^2}{2d}\right)$$

$$p.d = d + \frac{(h_t + h_r)^2}{2d} - d - \frac{(h_t - h_r)^2}{2d}$$

$$p.d = \frac{(h_t + h_r)^2 - (h_t - h_r)^2}{2d} = \frac{4h_th_r}{2d}$$

Phase angel due to path difference is given by

$$\alpha = \frac{2\pi}{\lambda}(p.d) = \frac{2\pi}{\lambda}\frac{4h_th_r}{2d} = \frac{4\pi h_th_r}{\lambda d} \qquad -4$$

Total phase difference between the direct wave and reflected wave is given by

$$\theta = \alpha + \beta$$

Where β is called the phase of the reflected signal due to reflection by the earth which is equal to 180^0.

$$\theta = \alpha + 180^0 \qquad -5$$

The total electric field strength at the receiving point is given by

$$E_R = E_0e^{j0} + kE_0e^{-j\theta} = E_0\left(1 + ke^{-j\theta}\right)$$

$$E_R = E_0\left(1 + k(cos\theta - jsin\theta)\right)$$

$$E_R = E_0\left((1 + kcos\theta) - jksin\theta\right)$$

$$|E_R| = E_0\sqrt{(1 + kcos\theta)^2 + k^2sin^2\theta}$$

$$|E_R| = E_0\sqrt{1 + k^2cos^2\theta + 2kcos\theta + k^2sin^2\theta} = E_0\sqrt{1 + k^2 + 2kcos\theta}$$

But

$$cos\theta = 2cos^2\frac{\theta}{2} - 1$$

$$|E_R| = E_0\sqrt{1 + k^2 + 2k\left(2cos^2\frac{\theta}{2} - 1\right)}$$

But for good conductor the reflection coefficient k equal to unity.

$$|E_R| = E_0\sqrt{1 + 1 + 2\left(2cos^2\frac{\theta}{2} - 1\right)} = E_0\sqrt{2 + 4cos^2\frac{\theta}{2} - 2}$$

$$|E_R| = 2E_0cos\frac{\theta}{2} \qquad -6$$

Substitute equation 5 in equation 6

$$|E_R| = 2E_0cos\left(\frac{\alpha + 180^0}{2}\right) = 2E_0sin\left(\frac{\alpha}{2}\right) \qquad -7$$

Substitute equation 4 in equation 7

$$|E_R| = 2E_0sin\left(\frac{4\pi h_t h_r}{2\lambda d}\right)$$

$$|E_R| \cong 2E_0\left(\frac{4\pi h_t h_r}{2\lambda d}\right) = E_0\frac{4\pi h_t h_r}{\lambda d}$$

But

$$E_0 = \frac{7\sqrt{P}}{d}$$

Where P is the power transmitted.

$$|E_R| = \frac{7\sqrt{P}}{d}\frac{4\pi h_t h_r}{\lambda d}$$

$$|E_R| = \frac{88\sqrt{P}\,h_t h_r}{\lambda d^2}$$

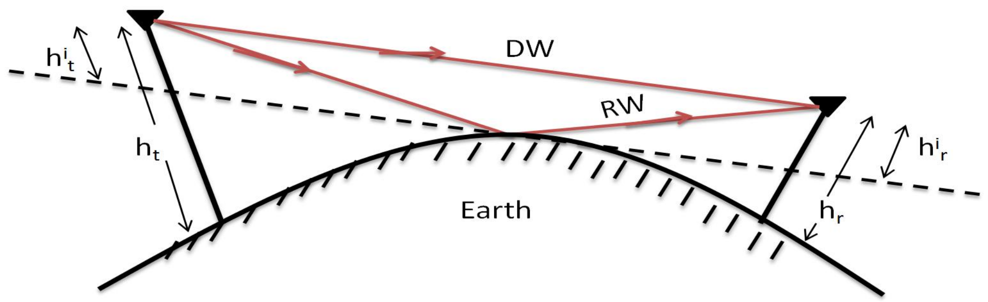
Where

P is the power transmitted
h_t is the height of the transmitting antenna
h_r is the height of the receiving antenna
λ is the wavelength
d is the distance between the transmitting and receiving antennas.

It can be observed that, the filed strength and distance are inversely proportional that is the field strength is decreases with increase in distance. Also it is observed that, the space wave field strength is a function of heights of the transmitting and receiving antenna. To increases the field strength, either the height of the transmitting antenna or receiving antenna or both can be increased.

Effect of earth's curvature:

The direct and reflected waves due to the curved earth are shown in the figure below.



Due to curvature of the earth:

(i) The effective and actual heights of the antennas are differ.
(ii) There is a change in number and location of maxima and minima.
(iii) The wave reflected by the ground diverges.
(iv) At large distances, for small incident angles, the direct and reflected waves are in phase opposition.

Absorption:

In very high frequency ranges, the rain attenuates the wave partly due to absorption and partly by scattering. This attenuation is a function of wavelength, permittivity, drop diameter and drop concentration. Serious attenuation is observed at $\lambda = 3$ cm for heavy rains and at $\lambda = 1$cm for moderate rains. High attenuation will occur at $\lambda = 1$ cm due to clouds and fog. Losses due to ice are less than the liquid water. As water content in a snow storm is quite small, and hence the attenuation caused by the snow is always small. Due to molecular interaction, absorption of energy takes place at certain wavelengths due to atmospheric gases like water vapor and oxygen.

SUPER REFRACTION, M-CURVES AND DUCT PROPAGATION:

Combination of scattering, refraction and reflection is known as super refraction or duct propagation. The refractive index of the atmosphere is given by

$$\mu = \sqrt{\varepsilon_r}$$

Modified refractive index is given by

$$N = \mu + \frac{h}{r}$$

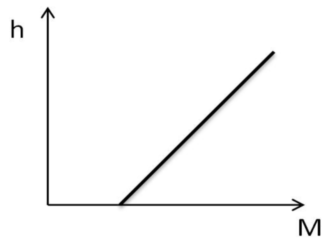
Where h is the height above the earth surface, r is the earth radius and ε_r is the relative permittivity of the atmosphere.

Excessive modified refractive index modulus is given by

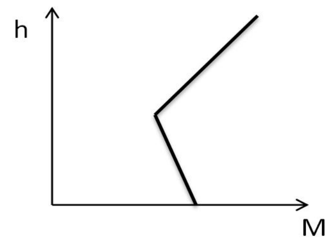
$$M = (N - 1)10^6$$

Variation of M with h represents the M-curves.

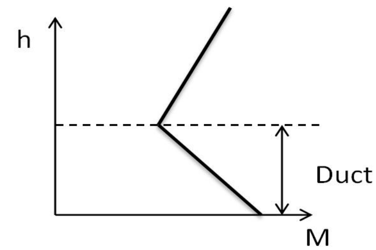
M-curves and duct formation is shown in the figure below.



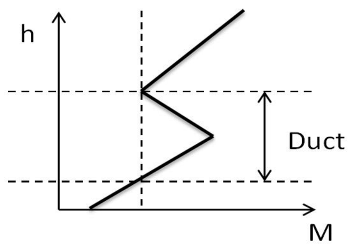
Fig(a): Standard atmosphere



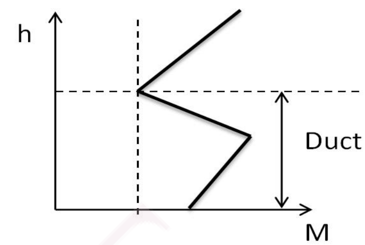
Fig(b): Temperature inversion



Fig(c): Ground based duct

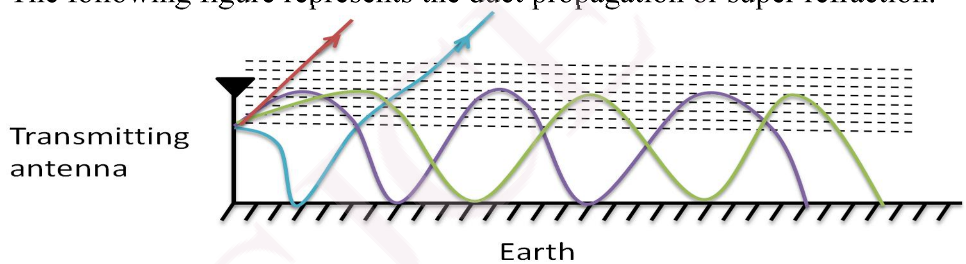


Fig(d): Elevated duct



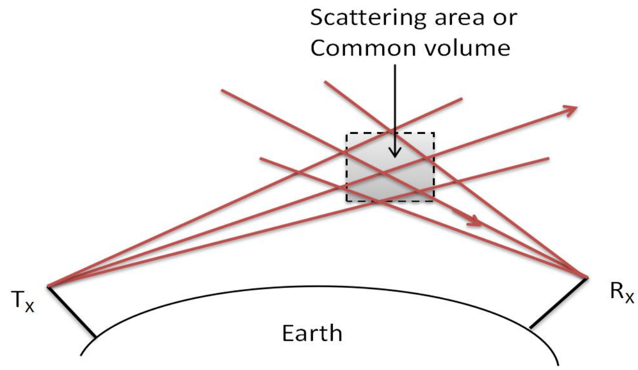
Fig(e): Ground based duct

Duct is nothing but a tube or channel. The ducts will form wherever the temperature inversion takes place. The following figure represents the duct propagation or super refraction.



SCATTERING PHENOMENA

Scattering is defined as the physical process in which the wave is forced to deviate in some other direction other than the intended direction due to presents of some objects or non uniformities in the atmosphere. Reception of signal far beyond the optical horizon in VHF and UHF range is possible due to the scatter propagation. Both troposphere and ionosphere are in continual state of turbulence. This gives rise to local variation of refractive index (μ) of the layer. The waves passing through such turbulent regions get scattered. The following figure represents the scattering phenomenon that takes place in troposphere or ionosphere. The scattered signal travels in different directions. Some signal may travel towards the receiver which is known as forward scattering, where as some signal travels back towards the transmitter which is known as back scattering. The scattering in troposphere will takes place when the frequency is greater than 500 MHz and the scattering in ionosphere will takes place when the frequency is in between 30 to 50 MHz. With troposcattering a maximum distance of 300 to 600 km can be covered where as with ionospheric scattering, a range up to 2000 km can be covered.



TROPOSPHERIC PROPAGATION

The scattering that takes place in troposphere can be used for the communication purpose. When the wave propagates from the transmitter to the receiver through the troposheric layer, then it is known as tropospheric propagation. The tropospheric propagation will occur due to the scattering or due to space waves. Tropospheric wave propagation is also called as space wave propagation. The fundamental equation (Friis transmission equation) for the space wave propagation or free space wave propagation is derived as follows:

The gain of the antenna under transmitting mode is defined as

$$G_T = \frac{Power\ density\ from\ the\ transmitting\ antenna}{Power\ density\ from\ the\ reference\ antenna} = \frac{P_D}{P_T/_{4\pi d^2}}$$

Where P_T is the power transmitted, d is the distance between the two antennas

$$Power\ density\ transmitted(P_D) = \frac{P_T G_T}{4\pi d^2}$$

When A_e be the effective area of the receiving antenna, then the received power (P_R) is given by

$$P_R = P_D \times A_e = \frac{P_T G_T}{4\pi d^2} \times A_e \qquad -1$$

But the relation between the gain (G_R) of the receiving antenna and its effective area (A_e) is

$$G_R = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{G_R \lambda^2}{4\pi} \qquad -2$$

Substitute equation 2 in equation 1

$$P_R = \frac{P_T G_T}{4\pi d^2} \times \frac{G_R \lambda^2}{4\pi}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

The above equation represents the fundamental equation for the free space propagation.

FADING AND PATH LOSS CALCULATIONS

Fading: A space wave signal or tropospheric wave signal often suffers from fading due to the variation of refractive index. Fading is defined as the reduction of signal amplitude or fluctuation of signal strength at the receiver due to effect of the propagating medium. The following figure represents the variation of signal strength due to the fading phenomena. Fading normally in Rayleigh nature. It can be classified in many ways. It can be fast fading or slow fading, single path or multi-path fading, short term or long term fading, etc. For fast and multi-path fading the duration is of the order of 0.01 second. To avoid the fading diversity techniques will be used.

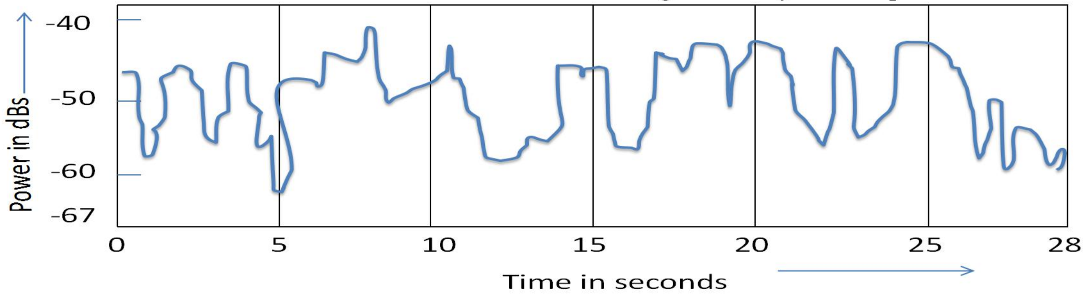


Fig: Fading phenomena

Path loss calculations: The fundamental equation for free space propagation is given by

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \qquad or \qquad P_R = \frac{P_T G_T G_R}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

In above equation, the term $\left(\frac{4\pi d}{\lambda}\right)^2$ is called loss factor because, due to increase of this factor the power received will be decreases. Therefore the path loss can be calculated from this factor.

$$Path\ loss = \frac{16\,\pi^2 d^2}{\lambda^2} = \frac{16\,\pi^2 d^2}{\left(\frac{c}{f}\right)^2} = \frac{16\,\pi^2 d^2 f^2}{c^2}$$

When d is in km, f is in MHz, then the above equation can be expresses in dB as

$$Path\ loss(in\ dB) = 32.45 + 20\log_{10}(f_{MHz}) + 20\log_{10}(d_{km})$$

SKY WAVE PROPAGATION

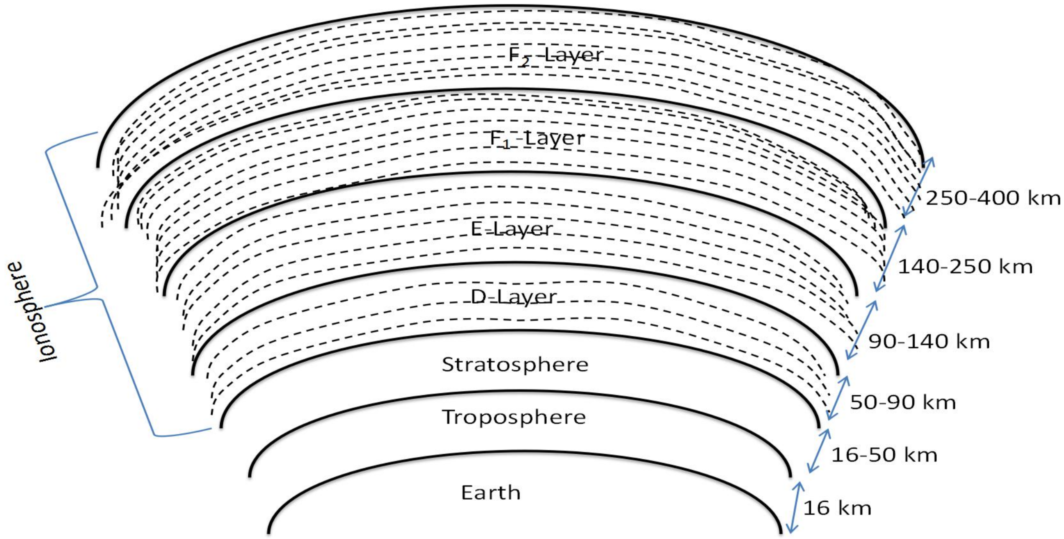
Introduction:

When the waves propagate through the Ionospheric layer of atmosphere then it is known as sky wave or ionospheric wave propagation. This propagation will be used for the frequency between 2 MHz to 30 MHz. It covers very long distance as compared with ground wave and sky wave propagation.

Structure of ionosphere:

The ionosphere is a region or layer above the earth and is composed of ionized layers. It is the highest layer of the atmosphere. The ionization is appreciable in this layer and hence the name ionosphere arises. The structure of ionosphere is shown in the figure below. The ionosphere consists of three layers such as D-layer, E-Layer and F-layer.

D-layer: It exists between the heights 50 to 90 km above the earth's surface. D-Layer is also known as Kennelly Heaviside layer. This layer will exist during the day time only where as in night times it disappears. The ionization in this layer is less because, it receives lee amount of solar energy. This layer will absorb the signal in the LF and MF ranges.



E-layer: This layer exists at a height of 90 to 140 km above the earth's surface with maximum density at 110 km. This layer is also known as Appleton layer. This almost constant with little diurnal and seasonal variations. It permits medium distance communication in LF and HF frequency bands. In E-layer there is one more layer called sporadic E-layer which exists occasionally. This layer results of an anomalous phenomena and falls under the category of irregular variations. The main reason for the existence of this layer is infiltration of charge carriers from the upper layers. This layer may not exist at all places. Also it do not present at all seasons.

F-Layer: It exists at heights of 150 to 400 km above the earth's surface. It is subdivided in to two layers such as F_1 layer and F_2 layer. F_1 layer will present at a height of 150 to 250 km where as F_2 layer will present at a height of 250 to 400 km. This F-layer will be used for long distance communication. During the night time both F_1 and F_2 layers combine and form as a single layer because of less solar energy during the night times.

Refraction and reflection of sky waves by ionosphere:

The basic principle of ionosphereic propagation is reflection but the phenomenon that takes place is refraction. To understand the refraction and reflection of sky waves, it is better to derive the equation for the refractive index of the ionosphere.

Let $E = E_m \sin\omega t$ be the electric field strength that enter in to the ionosphere. Due to this electric field, there is a force on the electrons present in the ionosphere. This force is given by

$$F_e = -eE \qquad -1$$

Where e is the charge of electron and F_e is the electric force.

Also from Newton's law the force is given by

$$F = ma = m\frac{dv}{dt} \qquad -2$$

Where 'm' is the mass of electron, 'v' is the velocity of electron and 'a' is the acceleration.

Equate equations 1 and 2

$$m\frac{dv}{dt} = -eE$$

$$dv = -\frac{eE}{m}dt$$

Take integration on both sides,

$$v = -\int \frac{eE}{m}dt = -\frac{e}{m}\int E_m \sin\omega t$$

$$v = \frac{eE_m cos\omega t}{m\omega}$$

$$v = \left(\frac{e}{m\omega}\right) E_m cos\omega t \qquad -3$$

If 'N' be the number of electrons per cubic meter, then the instantaneous electric current is given by

$$i_e = -Nev \qquad -4$$

Substitute equation 3 in 4

$$i_e = -Ne\left(\frac{e}{m\omega}\right) E_m cos\omega t = -\frac{Ne^2}{m\omega}\ E_m cos\omega t \qquad -5$$

The above current is called as inductive current. Beside this inductive current , there is additional current called capacitive current or displacement current due to the displacement of charge carriers. This current is known as capacitive current and is given by

$$i_c = \frac{dD}{dt} = \frac{d}{dt}(\varepsilon_0 E) = \frac{d}{dt}(\varepsilon_0 E_m \sin\omega t)$$

$$i_c = \varepsilon_0 E_m cos\omega t\omega \qquad -6$$

Total current is given by

$$i = i_c + i_e \qquad -7$$

Substitute equations 6 and 5 in equation 7

$$i = \varepsilon_0 E_m cos\omega t\omega\ -\frac{Ne^2}{m\omega}\ E_m cos\omega t$$

$$i = E_m cos\omega t\omega\ \left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right] \qquad -8$$

By comparing equations 6 and 8 we can say that the term $\left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right]$ is called effective dielectric constant (ε).

$$\varepsilon = \left[\varepsilon_0 - \frac{Ne^2}{m\omega^2}\right]$$

$$\varepsilon = \varepsilon_0\left[1 - \frac{Ne^2}{m\omega^2\varepsilon_0}\right]$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 - \frac{Ne^2}{m\omega^2\varepsilon_0}$$

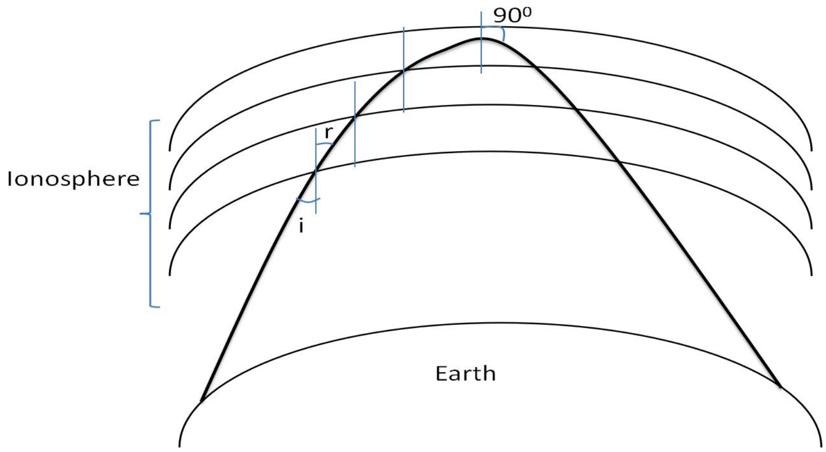
The refractive index is given by

$$\mu = \sqrt{\varepsilon_r} = \sqrt{1 - \frac{Ne^2}{m\omega^2\varepsilon_0}}$$

Substitute the values m = 9.107 X10^{-31} kg, e = 1.602 X 10^{-19} coulombs, ε_0 = 8.854 X 10^{-12} and ω = 2πf, Then

$$\mu = \sqrt{1 - \frac{81\ N}{f^2}}$$

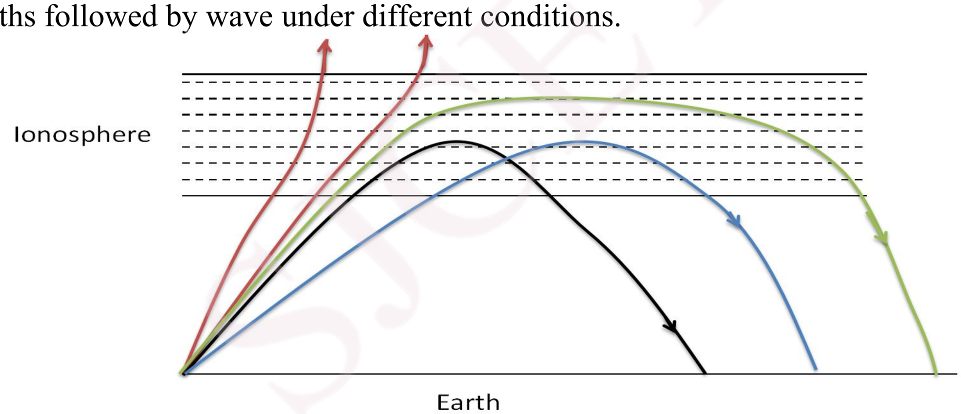
The above equation represents the refractive index of the ionosphere. The following figure represents the principle of refraction and reflection of sky wave by the ionosphere.



When the EM wave enters in the ionosphere, it undergoes refraction due to the variation of refractive index. The refractive index of upper layers will be less than the lower layers. Due to this variation of refractive index, the wave will bend as shown in the figure. The angel of refraction is go on increases as the wave travels through the layers of ionosphere. At a peak point the angle of refraction becomes 90^0, then onwards the wave travel in downward direction and finally it reaches the receiving point on the earth surface.

Ray path:

The path followed by the wave is termed as ray path. The following figure represents the different paths followed by wave under different conditions.



When the angle of incidence (i) is large, then the wave will reflected by the ionosphere and returned to the earth as indicated by ray 1.When the angle of incidence decreases, the penetration of wave in to the layer will increases as shown by the rays 2, 3 and 4. When the angle of incidence is very small, the waves will enter in to the outer region and they will not return to the earth surface as shown by the rays 5 and 6.

Critical frequency:

The critical frequency (f_c) is defined as the highest frequency that returns from the ionosphere layer at vertical incidence for that particular layer. Different layers will posses different values of critical frequency. The equation for the critical frequency is derived as follows:

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}}$$

But the critical frequency is defined for vertical incidence, that is i = 0^0, then the above equation becomes

$$0 = \sqrt{1 - \frac{81\,N}{f^2}}$$

$$0 = 1 - \frac{81\,N}{f^2}$$

$$\frac{81\,N}{f^2} = 1$$

$$f^2 = 81\,N$$

$$f = f_c = \sqrt{81\,N} = 9\sqrt{N} = 9\sqrt{N_{max}}$$

Where N_{max} is the maximum electron density of the ionospheric layer.

MUF (Maximum Usable Frequency):

MUF is abbreviated as Maximum Usable Frequency and is defined as the highest frequency that returns from the ionosphere layer at some angle of incidence other that the vertical incidence. Major difference between the critical frequency and MUF is, the critical frequency will be defined with respect to vertical incidence where as MUF will be defined with respect to some angle of incidence other than the vertical incidence. The equation for the MUF is derived as follows:

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}}$$

To have reflection from the ionosphere the angel of refraction r must be equal to 90^0.

$$\frac{\text{Sin}\, i}{\sin 90^0} = \sqrt{1 - \frac{81\,N}{f^2}}$$

$$\sin i = \sqrt{1 - \frac{81\,N}{f^2}}$$

$$sin^2 i = 1 - \frac{81\,N}{f^2}$$

$$\frac{81\,N}{f^2} = 1 - sin^2 i = cos^2 i$$

$$f^2 = \frac{81\,N}{cos^2 i} = \frac{f_c^2}{cos^2 i}$$

$$f = f_{MUF} = \frac{f_c}{\cos i} = f_c \sec i$$

The above equation represents the MUF. This equation is specifically called as secant law because it is in terms of secant.

LUF (Lowest Usable Frequency):

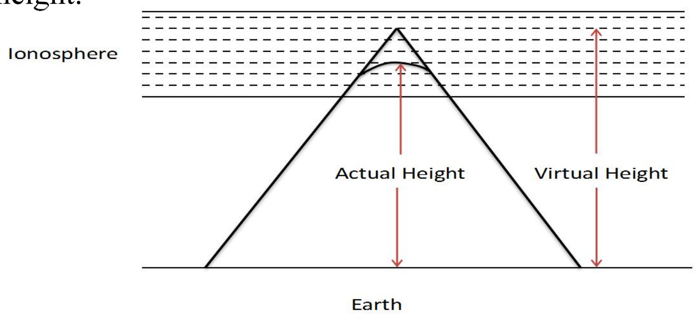
Lowest usable Frequency (LUF) is defined as the frequency below which the entire signal strength will be absorbed by the ionospheric layer. MUF gives the upper limit on the usable frequency where as LUF gives the lower limit on the usable frequency.

OF (Optimum Frequency):

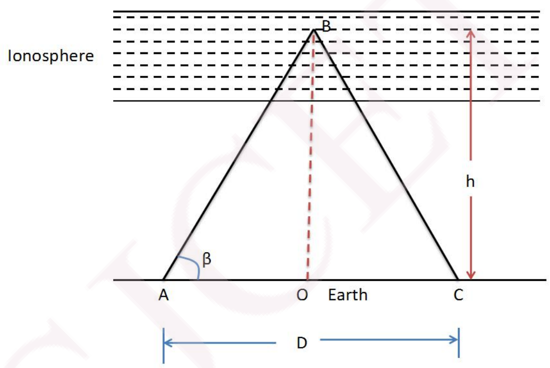
MUF and LUF give the upper and lower limit on the usable frequency but they may not be used to select the practical operating frequency. The reason for this is the MUF and LUF will change with heights of the layers. Therefore it is required to define on more frequency called optimum frequency (OF) or optimum working frequency (OWF) for the selection operating frequency. The Optimum Frequency (OF) is 85% of MUF.

Virtual height and skip distance:

Virtual Height: It may be defined as "the height to which a short pulse of energy sent vertically upward and traveling with the speed of light would reach taking the same two-way travel time as does the actual pulse reflected from the ionospheric layer". The following figure represents the virtual height ant physical height (Actual height). The virtual height of the ionospheric layer is larger than the actual height.



The virtual height of the flat earth shown in figure below



The virtual height of the flat earth shown is given by

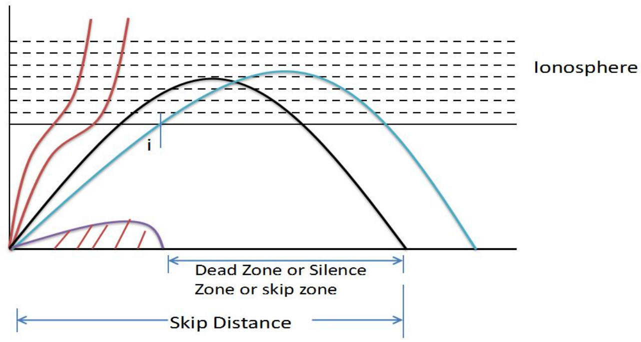
$$h = \frac{D\ tan\beta}{2}$$

where D is the skip distance, β is the elevation angle. The virtual height of the curved earth is given by

$$h = \frac{cT}{2}$$

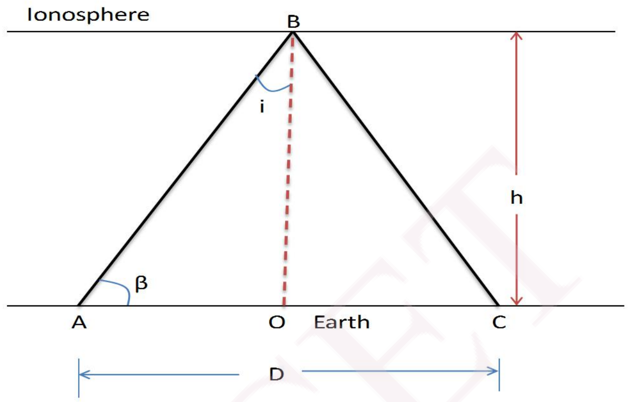
where c is the velocity of light and T is the two-way transit time of pulse.

Skip distance: The following figure represents the skip distance. The skip distance is defined as the minimum distance from the transmitter at which sky wave of given frequency is returned to earth by the ionosphere. It is also defined as minimum distance within which, the sky wave of given frequency fails to be reflected back.



Relation between muf and skip distance:

For flat earth, the relation in between the MU=F (f_{MUF}) and skip distance (D) is derived from the following figure.



From Δ OAB,

$$\cos i = \frac{h}{\sqrt{h^2 + \frac{D^2}{4}}} \qquad - 1$$

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81\,N}{f^2}} = \sqrt{1 - \frac{81\,N_{max}}{f_{MUF}^2}}$$

But

$$81\,N_{max} = f_c^2$$

And

Angel of refraction r = 90^0 to have the reflection by the ionosphere

$$\frac{\sin i}{\sin(90)} = \sqrt{1 - \frac{f_c^2}{f_{MUF}^2}}$$

$$\sin i = \sqrt{1 - \frac{f_c^2}{f_{MUF}^2}}$$

$$sin^2 i = 1 - \frac{f_c^2}{f_{MUF}^2}$$

$$\frac{f_c^2}{f_{MUF}^2} = 1 - sin^2 i = cos^2 i \qquad -2$$

Substitute equation 1 in equation 2

$$\frac{f_c^2}{f_{MUF}^2} = \left(\frac{h}{\sqrt{h^2 + \frac{D^2}{4}}} \right)^2$$

$$\frac{f_{MUF}^2}{f_c^2} = \frac{4h^2 + D^2}{4h^2}$$

$$\frac{f_{MUF}}{f_c} = \sqrt{\frac{4h^2 + D^2}{4h^2}}$$

$$f_{MUF} = f_c \sqrt{\frac{4h^2 + D^2}{4h^2}} = f_c \sqrt{1 + \left(\frac{D}{2h}\right)^2}$$

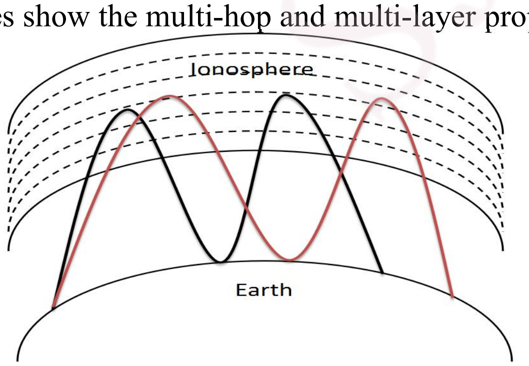
Skip distance is given by

$$D = 2h \sqrt{\frac{f_{MUF}^2}{f_c^2} - 1}$$

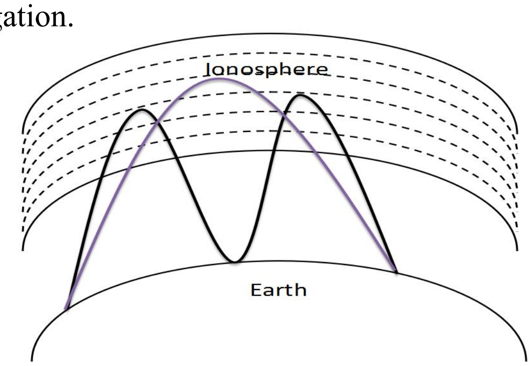
The above equation represents the relation between the MUF and skip distance for flat earth or plane earth.

MULTI-HOP PROPAGATION

When a sky travels from the transmitting antenna to the receiving antenna with more than one reflection between the inospheric layer and earth surface then it is known as Multi-Hop propagation. With multi-hop propagation a wave can travel very long distance. The following figures show the multi-hop and multi-layer propagation.



Fig(a): Multihop propagation



Fig(b): Multilayer propagation

In multi-hop propagation, a wave will utilize a particular layer to undergo reflection where as in multi-layer propagation; a wave will utilize more than one layer for its propagation.

ENERGY LOSS IN IONOSPHERE

Even though the gas pressure in the ionosphere is very low, the vibrating electrons collide with gas molecules from time to time. Due to this collision, the kinetic energy (acquired from EM waves) of electrons will lost. The amount of this loss depends on the gas pressure, velocity of vibration, likelihood of collision and frequency of collision. Most of the absorption loss takes place at a lower edge of the ionized region, where the atmospheric pressure is greater. At high frequency, the energy loss due to collision occurs mainly just below the E layer where product of collosional frequency and electron density is maximum.

SUMMARY OF WAVE CHARACTERISTICS IN DIFFERENT FREQUENCY RANGES

Based on the frequency of operation, the waves have been characterized in to different types and they are explained below:

VLF Waves:

- Range of VLF (Very Low Frequency) waves spreads over 3 KHz to 30 KHz.
- These waves cannot be used for conventional communication.
- These are called long waves because of their large wavelength and they can travel up to very long distance.
- These waves can penetrate deeper in to a sea as well as the earth.

20 KHz – 100 KHz:

- This range encompasses part of the VLF band part of LF (Low Frequency) band.
- In this range, the ground waves have relatively low attenuation.
- Signals in this range are stronger at night than day.
- Signals are stronger in winter than in summer.

100 KHz – 535 KHz:

- This range encompasses part of the LF band and part of MF (Medium Frequency) band.
- Ground waves attenuate more rapidly as the frequency are raised above 100 KHZ.
- Ionospheric losses tend to be high in day time but remain low at night.
- Night–time communication for long distances by sky wave are reliable.

535 KHz – 1600 KHz:

- This range is segment of MF band.
- Ground waves attenuate very rapidly.
- The range of frequencies to be used depends on the given set of conditions.
- Almost all long distance communications use ionospheric reflections.

VHF Waves:

- These waves are called Very High Frequency waves or simple called as short waves.
- All modes of propagation is possible but with different amount of attenuation.
- These waves are capable of passing through the ionosphere as direct wave.

UHF and SHF Waves:

- These waves occupies the frequency range of Ultra High frequency (UHF) and Super High Frequency (SHF).
- Communication for long distances through troposheric waves.
- Diffraction in this range is negligible.

EHF Waves:

- These waves occupy the band of Extremely High Frequency (EHF).
- The wavelength of these waves is in the order of millimeter.
- Rain, fog, hail, and other forms of precipitation particles responsible for marked absorption.
- Heavy rain and dense fog will completely stop the communication.
- Strong molecular absorption by tropospheric gases especially water vapor and oxygen.

Sub-millimetric and Optical Waves:

- Can propagate only as ground and direct wave.

- Atmospheric refraction causes bending of path.
- Heavy rain and dense fog will completely stop the communication.
- Well suited for space communication outside the troposphere.

PROBLEMS:

(1) Calculate the distance beyond which the earth's curvature is to be accounted at frequency of (a) 100 KHz (b) 1 MHz (c) 10 MHZ.

Ans: f = 100 KHz = 0.1 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{0.1^{1/3}} = 107.75\ miles$$

(a) f = 1 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{1^{1/3}} = 50\ miles$$

(b) f = 10 MHz

$$d = \frac{50}{(f_{MHz})^{1/3}} \quad in\ miles = \frac{50}{10^{1/3}} = 23.21\ miles$$

(2) Obtain the roughness factor at 3 MHz for an earth having σ = 0.5 with θ = 30⁰. Calculate the ratio of roughness factors for the same earth and same θ if the frequency is doubled.

Ans: Given data: Frequency (f) = 3 MHz

Wavelength (λ) = c/f = $3X10^8/3X10^6$ = 100 m

Standard deviation (σ) = 0.5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 0.5 \times \sin(30)}{100} = 0.031415927$$

When the frequency is doubled, Frequency (f) = 2X3 MHz = 6 MHz

Wavelength (λ) = c/f = $3X10^8/6X10^6$ = 50 m

Standard deviation (σ) = 0.5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 0.5 \times \sin(30)}{50} = 0.06282$$

(3) Evaluate the roughness factors for the earth at 10 MHz if σ = 5 for θ equal to (a) 30^0 (b) 45^0 (c) 60^0

Ans:

(a) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f = $3X10^8/10X10^6$ = 10 m

Standard deviation (σ) = 5

Angle of incidence (θ) = 30^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin(30)}{10} = 3.141$$

(b) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f = $3X10^8/10X10^6$ = 10 m

Standard deviation (σ) = 5

Angle of incidence (θ) = 45^0

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin(45)}{10} = 4.442$$

(c) Given data: Frequency (f) = 10 MHz

Wavelength (λ) = c/f =3X10⁸/10X10⁶ = 10 m
Standard deviation (σ) = 5
Angle of incidence (θ) = 60⁰

$$Roughness\ factor\ (R) = \frac{4\pi\sigma sin\theta}{\lambda} = \frac{4\pi \times 5 \times \sin{(60)}}{10} = 5.44$$

(4) The transmitting and receiving antennas with respective heights of 49 m and 25 m are installed to establish communication at 100 MHz with transmitted power of 100 watts. Determine the LOS range and received signal Strength.

Ans: Given data: $\text{Height of the transmitting antenna}(h_t) = 49\text{ m}$

$$\text{Height of the receiving antenna}(h_r) = 25\text{ m}$$
$$\text{Frequency(f)} = 100\text{ MHz}$$
$$\text{Wavelength}(\lambda) = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3\text{ m}$$
$$\text{Power transmitted(P)} = 100\text{ watts}$$

LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km$$
$$d = 4.12\left[\sqrt{49} + \sqrt{25}\right]km = 4.12[7 + 5]km = 49.44\ km$$

The received field strength is given by

$$|E_R| = \frac{88\sqrt{P}\ h_t h_r}{\lambda d^2} = \frac{88\sqrt{100} \times 49 \times 25}{3 \times (49.44 \times 1000)^2} = 1.47\ V/m$$

(5) Calculate the maximum distance at which signal from transmitting antenna with 144 m height would be received by the receiving antenna of 25 m height.

Ans: Given data: $\text{Height of the transmitting antenna}(h_t) = 144\text{ m}$

$$\text{Height of the receiving antenna}(h_r) = 25\text{ m}$$

Maximum distance or LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km = 4.12\left[\sqrt{144} + \sqrt{25}\right]km = 4.12[12 + 5]km$$
$$d = 70.04\ km$$

(6) A transmitting antenna of 100 m height radiates 40 kW at 100 MHz uniformly in azimuth plane. Calculate the maximum LOS range and strength of the received signal at 16 m high receiving antenna at a distance of 10 km. At what distance would the signal strength reduces to 1 mV/m.

Ans: Given data: $\text{Height of the transmitting antenna}(h_t) = 100\text{ m}$

$$\text{Height of the receiving antenna}(h_r) = 16\text{ m}$$
$$\text{Frequency(f)} = 100\text{ MHz}$$
$$\text{Wavelength}(\lambda) = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3\text{ m}$$
$$\text{Power transmitted(P)} = 40\text{ kW}$$
$$\text{Distance(d)} = 10\text{ km}$$

LOS range is given by (including effective earth's radius)

$$d = 4.12\left[\sqrt{h_t} + \sqrt{h_r}\right]km = 4.12\left[\sqrt{100} + \sqrt{16}\right]km = 4.12[10 + 4]km = 57.68\ km$$

The received field strength is given by

$$|E_R| = \frac{88\sqrt{P}\ h_t h_r}{\lambda d^2} = \frac{88\sqrt{40 \times 10^3} \times 100 \times 16}{3 \times (10 \times 1000)^2} = 98.36\ mV/m$$

The distance at which the field strength reduces to 1 mV/m is

$$|E_R| = \frac{88\sqrt{P}\,h_t h_r}{\lambda d^2}$$

$$d = \sqrt{\frac{88\sqrt{P}\,h_t h_r}{|E_R|\lambda}} = \sqrt{\frac{88\sqrt{40\times10^3}\times100\times16}{1\times10^{-6}\times3}} = 96.88\,km$$

(7) A directional antenna with 10 dB gain radiates 500 watts. The receiving antenna at 15 km distance receives 2 μW. Find the effective area of the receiving antenna. Assume negligible ground and ionospheric reflections.

Ans: Given data: Gain of transmitting antenna$(G_T) = 10$ dB

Gain without dB is given by

$$10 = 10\log(G_T)$$

$$G_T = 10$$

$$\text{Power transmitted}(P_T) = 500\text{ W}$$

$$\text{Distance}(d) = 15\text{ km}$$

$$\text{Power received}(P_R) = 2\ \mu\text{W}$$

We know that,

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

$$P_R = \frac{P_T G_T}{4\pi d^2} A_e$$

$$A_e = \frac{P_R\,4\pi d^2}{P_T G_T} = \frac{2\times10^{-6}\times4\pi\times(15\times1000)^2}{500\times10} = 1.13\,m^2$$

(8) Find the basic path loss for communication between two points 3000 km apart at a frequency of 3 GHz.

Ans: Given data: Distance$(d) = 3000$ km

$$\text{Frequency}(f) = 3\text{ GHz}$$

$$Path\ loss = 32.45 + 20\log_{10} f_{MHz} + 20\ \log_{10} d_{km}$$

$$Path\ loss = 32.45 + 20\log_{10}(3\times1000) + 20\ \log_{10}(3000) = 171.534\text{ dB}$$

(9) Calculate the skip distance for flat earth with MUF of 10 MHz if the wave is reflected from a height of 300 km where the maximum value of n is 0.9

Ans: Given data: MUF$(f_{MUF}) = 10$ MHz

$$\text{Height of the layer}(h) = 300\text{ km}$$

$$\text{Refractive index}(\mu\text{ or n}) = 0.9$$

$$\mu = \sqrt{1 - \frac{81\,N_{max}}{f^2}}$$

$$N_{max} = \frac{(1-\mu^2)f^2}{81} = 23.45\times10^{10}$$

Critical frequency is $f_c = 9\sqrt{N_{max}} = 9\sqrt{23.45\times10^{10}} = 4.36\,MHz$

Skip distance is given by

$$D = 2h\sqrt{\frac{f_{MUF}^2}{f_c^2} - 1} = 2\times300\times10^3\sqrt{\frac{(10\times10^6)^2}{(4.36\times10^6)^2} - 1} = 3916.2\,km$$

(10) The critical frequencies at an instant observed for E, F_1 and F_2 layers were found to be 3, 5 and 9 MHz. Find the corresponding concentration of electrons in these layers.

Ans: The critical frequency is given by

$$f_c = 9\sqrt{N_{max}}$$

Then,

$$N_{max} = \frac{f_c^2}{81}$$

For E layer, f_c = 3 MHz,

$$N_{max} = \frac{(3 \times 10^3)^2}{81} = 0.111 \times 10^{12}$$

For F_1 layer, f_c = 5 MHz,

$$N_{max} = \frac{(5 \times 10^3)^2}{81} = 0.3086 \times 10^{12}$$

For F_2 layer, f_c = 9 MHz,

$$N_{max} = \frac{(9 \times 10^3)^2}{81} = 10^{12}$$